# EVALUATING THE ROLE OF PROGRAMMING COMMON STIMULI AND PRIOR SUB-SKILL FLUENCY IN PROMOTING GENERALIZATION OF BASIC MATH FACTS

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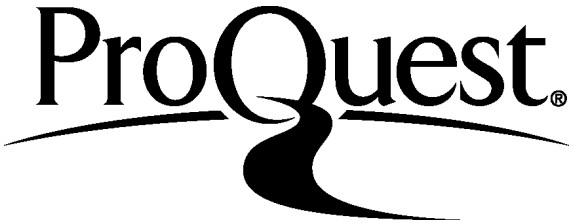
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Abstract: The current project involves a two-part study examining the role of programming common stimuli and prior sub-skill fluency in promoting the response generalization of basic math fact computation. Study 1 involved a single-case alternating treatments design with a second grade class to examine the effect of programming common stimuli by observing differential treatments of Cover Copy Compare using fact family triangles (i.e. programming common stimuli) and an Explicit Timing Procedure in basic addition and their generalization to basic subtraction. This examination compared the differences in these two treatments (1) in increasing learning rates of the target skill of single-digit addition, (2) in promoting generalization to single-digit subtraction from fluency building in addition in digits correct per minute (DCPM), (3) in promoting generalization to subtraction when paired with procedural strategies, and (4) examining the interaction of procedural strategies with prior fluency levels of addition. Results indicated that the Explicit Timing intervention produced faster growth rates, but Cover Copy Compare resulted in higher levels of generalization to subtraction when paired with an instructional procedure. Follow-up analyses also indicated that generalization to subtraction skills only occurred if students achieved mastery levels of fluency in addition. The second study extends the findings of the first study through a large-N analysis of the interaction between prior basic multiplication fluency and the use of application strategies in promoting the generalization of basic division. Specifically, a repeated measures linear mixed model design was used to compare growth differences in division DCPM between students who showed mastery levels of fluency (>35 DCPM), instructional levels (20-34 DCPM), and low levels (<19) in basic multiplication on both a trained and untrained problem set. Analyses using growth curve modeling indicated that the high fluency group had significantly higher generalization effects than the other two groups, and all groups had significantly higher generalization effects on the trained problem set (of higher fluency) over the untrained problem set. Overall project results provide further support that fluency in prior sub-skills is necessary but not sufficient in promoting generalization, and programming common stimuli may be a possible generalization technique to apply to academic intervention.



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# CHAPTER I

## INTRODUCTION

America is currently facing many crises as a nation. While historically conceptualized as a world leader, the United States has recently become idle in its economy and education. The palpable association between economy and education/skill development (Hanushek & Woessmann, 2008) indicates that U.S. educational deficits may promote future problems (Card, 1999). This association is especially seen regarding mathematics (Peterson, Lanstra-Anadon, Hanushek, & Woessman, 2011). Harvard's Program on Educational Policy and Governance found that in 2011, U.S. high-school graduates possessed only a 32% proficiency rate in mathematics (Peterson et al., 2011). These deficits are likely effects of earlier educational difficulty. These findings are further supported by The National Center for Education Statistics (NCES), which reported that only 35% of eight-grade students and 40% of fourth-grade students were meeting proficiency levels in math.

This drop in proficiency through the school years indicates that early identification, remediation, and skill development is crucial for meeting our students' needs. Several initiatives have been implemented in an attempt to do so. Implementation of No Child Left Behind (PL 107-110) has promoted the service of "high-quality education and reach, at a minimum, proficiency on challenging state academic achievement standards and state academic assessments" (p. 1439), while holding

educators "accountable for improving the academic achievement of all students" (p.1440). These focuses have highlighted the need for evidence-based practices (IDEIA, 2004) and frequent progress monitoring (Gresham, 2004) through the vehicle of Response to Intervention (RTI) (Fuchs, 2003; Gresham, 2005; Heartland Area Education Agency, 2007).

To accent the national deficit in mathematics, the National Mathematics Advisory Panel (NMAP, 2008) was formed to determine research based practices in math education and outlined steps for systematic educational improvement. The panel affirmed the importance of early education and remediation of foundational skills to prevent future deficits. To do so, the panel recommended that instruction result in students being able to fluently respond to basic facts problems. The panel also stated that students learn relevant procedures to support the development of "conceptual understanding". Finally, the panel suggested that teachers and programs utilize findings from research and evidence-based practice. Although the agenda has been set politically, and educators as well as school psychologist agree in their importance, there is less agreement regarding the vehicles required to achieve these goals (Cozza, Foley, & Laboranti, 2009; Shapiro, 2011).

Behavioral approaches in education have been less popular than models focusing on constructivism and internal experiential learning (Larochelle, Bednarz, & Garrison, 1998). With increasing accountability, the call for research-supported practice has given greater respect to the behavioral sciences with more pragmatic foundations. Skinner (1953) was a pioneer in the analysis of human behavior. This behavioral perspective focuses on how specific environmental contingencies directly contribute to behavior change. Environmental contingencies in education, such as praise or instructional

techniques, can empower educators by giving them control over their student's success (Lindsley, 1972).

The Instructional Hierarchy (IH;Haring & Eaton, 1978) has been widely used to guide empirically-supported intervention research in academics. Under this hierarchy, skill mastery requires sequential learning through accuracy, fluency, and generalization/adaptation. The first step towards mastery requires initial *acquisition* of a skill or behavior. This is the academic representation of stimulus control. Following accurate responding is the need to be *fluent* in the behavior, in which responding can occur quickly and with ease. Once fluency is attained, an individual must be able to *generalize* and *adapt* the behavior across new situations for it to be fully amalgamated into their behavioral repertoire. Generalization can be conceptualized as using a different response from the response that was trained (i.e. response generalization), responding in a setting or situation that was not under training (i.e. setting/situation generalization), or maintaining a trained response over time (i.e. time generalization) (Baer, Wolf, & Risley, 1968).

Using this IH as a template, one can differentially structure instruction to accommodate learning in each stage. For instance, accuracy (i.e. stimulus control) requires instruction that includes three-term contingency learning trials with antecedents, opportunities to respond, and consequences/feedback (Albers & Greer, 1991). Alternatively, instruction to increase fluency requires repeated drill and/or practice with reinforcement (Skinner et al., 1996).

It is hypothesized that generalization can most efficiently be programmed once a behavior can be fluently completed. While this may spontaneously occur under a train and hope situation (Stokes & Baer, 1977), generalization is often difficult to achieve and

may require active technologies. In addition to a train and hope method, Stokes & Baer (1977) identified eight additional technologies to promote generalization. These include (1) sequential modification, (2) introduce to natural maintaining contingencies, (3) train sufficient exemplars, (4) train loosely, (5) use indiscriminable contingencies, (6) train to generalize, (7) mediate generalization, and (8) program common stimuli. While all of these techniques hold empirical evidence across a variety of behaviors, little research has analyzed how these technologies can be systematically used to promote generalized academic responding. Two of these technologies are inherently tied to the assumptions laid out in the IH, indicating useful application in academics. Both "train and hope" and "program common stimuli" work from the assumption that generalization occurs after some finite amount of variety between known associations. Train and hope eventually achieves generalization naturally, while programming common stimuli does so systematically. This infers that responding fluently to known associations will allow for easier responding to similar-yet-unknown associations more frequently-and-often (Skinner et al. 1996). While this infers a sequential ordering of generalization programming after continuous "drill", it may be that generalization programming should also be repeatedly "practiced" (Haring & Eaton, 1978). Therefore, the use of programming common stimuli within various stages of the IH may create a more efficient route towards generalization.

The IH is a meaningful guide in matching student responding to instructional practices, but the methods of research contribute to further establishing such a guide. Specifically, instructional effectiveness (i.e. level change) has been highly valued as a main outcome, but could neglect important findings. Learning problems are better defined as a "failure to learn fast enough" rather than "failure to learn" (Skinner et al.,

1996). In the same regard, we may better understand and evaluate evidence-based practices by examining learning rate (learning/time) rather than overall learning. The paradigm change that values efficiency has better utilized more sensitive fluency measures for learning indicators and has allowed researchers to identify a conglomerate of instructional variables that increase learning while holding time constant.

Instructional efficiency can also be accomplished by serving more students in less time. Therefore, evidence-based interventions in class-wide environments should be a focus for school psychologists and further analyzed (Shapiro, 2004). Class-wide evidence-based interventions remediate many individual problems simultaneously, thus preventing future problems (Hawkins, 2010; Skinner, 2008), increase positive behavior universally, and improves educational decision making (Elliott, Turco, & Gresham, 1987; Gresham, 2004; Rathvon, 1999).

There have been many evidence-based interventions shown to remediate mathematic skills. Two prominent interventions are Cover Copy Compare (CCC) and Explicit Timing (ET). Cover Copy Compare is a self-managed intervention composed of repeating complete yet efficient learning trials (Skinner, McLaughin, & Logan, 1997). With CCC, students are repeatedly required to (1) look at a presented model (e.g., computation problem), (2) cover the model, (3) complete the required response in the absence of the model, and (4) check their answer with the model.

Cover Copy Compare has a comprehensive research base to confirm its utility across a variety of populations, skills, and environments (Joseph et al., 2012; Poncy & Skinner, 2011; Skinner, Bamberg, Smith, & Powell, 1993; Skinner, Ford, & Yunker, 1991). Cover Copy Compare effectively enhances both accuracy and fluency (Skinner, Turco, Beatty, & Rasavage, 1989). However, it also has been shown to be less time

efficient than other interventions (Poncy, Skinner, & Jaspers, 2007). This may be due to CCC's use of complete learning trials, which may limit response rates.

Explicit Timing (ET) is another empirically validated intervention for increasing math fluency. Because fluency is best achieved by engaging in efficient drill and practice (Haring & Eaton, 1978), ET takes direct advantage of this by simply encouraging fast responding of a targeted behavior by overtly timing it. Research has consistently shown that timing individuals while directly informing them to respond quickly significantly increases response rates (Houten & Thompson, 1976; Miller, Hall, & Heward, 1995). Repeating this contingency over time significantly increases fluency performance. Increased fluency from ET has been seen across race/ethnicity, math skill/difficulty, and intelligence levels (Evans-Hampton, Skinner, Henington, Sims, & McDaniel, 2002; Rhymer, Henington, Skinner, & Looby, 1999; Rhymer, Skinner, Henington, D'Reaux, & Sims, 1998; Rhymer et al., 2002).

Previous research with CCC and ET has validated both as empirically-supported fluency-building interventions; yet only one study has directly compared the two in this regard. Codding et al. (2007b) compared these two interventions through a randomized group design. Their results showed that there were no significant simple main effect differences between CCC and ET in increasing digits correct per minute (DCPM). However, differences were seen between the two interventions when interacting with initial fluency levels. Specifically, results showed that CCC was superior to ET for increasing fluency rates of individuals with initial fluency rates in the frustrational range, while ET was superior for increasing fluency levels when initial fluency rates were in the instructional range.

Studies evaluating math interventions generally have used DCPM as the dependent measure. Yet just as effectiveness can hide useful information compared to efficiency outcomes, measuring learning rate alone may also hide other important effects. Research empirically validating treatments need to also examine the effects of generalized responding. Rather, "learning is multivariate" (Cronbach & Snow, 1977, p.116), and any worthwhile treatment may affect learners in more than one way (Stevens, 2009). For example, one treatment that explicitly builds one skill but also promotes generalization to another related skill results in more overall learning than a treatment that only builds one skill. However, this cannot be empirically evaluated without the use of multiple criterion measures. Targeting generalized academic responding in this way has been done to a very limited extent, and more research in this area in needed.

Some research has found correlational data linking fluency to generalization (Lin & Kubina, 2005; Van Der Heyden & Burns, 2008; VanDerHeyden & Burns, 2009), but very few studies have experimentally examined generalization of math skills. Two studies (Codding et al., 2009a; Codding, Eckert,Fanning, Skiyko, & Solomon, 2007b) have found some evidence of generalization using a train and hope approach. Train and hope was seen as more successful in two additional studies (Codding, Archer, & Connell, 2010; Codding, Chan-Iannetta, Palmer, & Lukito, 2009b). The results from these four studies determine that generalization under train and hope is more likely to occur if the trained skill is topographically similar to the generalized outcome.

Only one study has examined generalization of math skill through active procedures. Poncy, Duhon, Lee, and Key (2010) utilized knowledge in the IH to examine generalization from addition to related subtraction skills using procedural instruction after addition skills were at fluent levels. Their attempts to systematically program

generalization were unsuccessful. Their results indicate that (1) the difference between the trained and generalized skill was too great and (2) procedural fluency as well as declarative fluency may be needed for generalization to occur.

Based on the IH and generalization research targeting the train and hope phenomena, it is supported that fluency plays an important role. However, what is less known are the mediating variables that need to be incorporated into fluent responding to reliably promote generalization. Several have been inferred. First, procedures like programming common stimuli might differentially effect generalization depending on when and with what stimuli it is practiced. Because topographical similarity is a strong predictor of generalization, incorporating common stimuli into the training environments that are shared in generalized outcomes may promote generalization by targeting functional similarity rather than topographical. Secondly, such procedures may need to become as fluent as declarative knowledge. However, this might also depend on the conditions in which declarative fluency was built. Thirdly, because generalization may depend on the similarity between trained and generalized outcomes, these effects should be analyzed across skills.

This dissertation analyzes two primary hypotheses across two studies targeting computation fluency. The first hypothesis is that programming common stimuli will lead to greater rates of generalization. The second hypothesis is that high levels of fluent responding are critical for students to generalize academic skills. The first study used a single-case alternating treatments design to examine the effect of programming common stimuli by observing differential treatments of CCC using fact family triangles and traditional ET. This examination compared the differences of these two treatments (1) in increasing learning rates of the target skill of single-digit addition, (2) in promoting

generalization to single-digit subtraction from fluency building in addition, and (3) in promoting generalization to subtraction using procedural strategies, and (4) examining the interaction of procedural strategies with prior fluency level of addition. The second study extends the findings of the first study through a large-N analysis of the interaction between prior basic multiplication fluency and the use of application strategies in promoting the generalization of basic division.

# CHAPTER II

# REVIEW OF THE LITERATURE

## Research and Education

The new state of affairs in education stressing accountability has allowed for more research backing to become more than lip service. More empirical models of educational psychology, such as the behavior analytic perspective, have taken the initiative in contributing to evidence-based practice. Behavior analytic perspectives focus on reliability of effect though discrete and objective measurement of behavior (Baer, Wolf, & Risley, 1968). Empirical focus is put on the consistency of behavior change across environmental conditions, rather than nonfunctional associations determined through traditional test theory. In this pragmatic approach to instruction, the student behaviors (e.g., academic responding) are the unit of analysis, rather than developmental constructs in which no functional relationship can be determined. Assessment employing this method is thus directly linked to treatment decisions and outcomes, possessing "treatment validity" (Fuchs & Fuchs, 1998).

Traditionally, research designs utilizing these behavioral assumptions have not been widely utilized in evaluating the efficacy of psychological and educational treatment. This has occurred for two reasons. First, experimental designs containing large sample sizes promote external validity (i.e. applicability of findings to a wide variety of settings and populations). Secondly, totally-experimental designs also promote high

internal validity through randomization. Because randomization is assumed to evenly distribute effects of variables other than the independent variable (i.e. error) across groups, differences in groups can be attributed to IV effects (Gay, Mills, & Airasian, 2009).

This outcome-focused movement has also revived the evaluation and use of single-case research to promote best practices (Kratochwill & Levin, 2010). While external validity is important, the prevalent needs in serving special education populations require evidence-based research that is applicable to discrete populations with idiosyncratic problems (Wolery, 2013). Single-case research is ideal in complementing this need. Using reliability of effect found through single-case research provides higher internal validity by showing all subjects' direct effects under analysis, especially when utilizing within-subject controls. Alternatively, group data can involve "non-responders" and can thus hide reliability of effect at an idiosyncratic level. It is also possible to improve external validity in single-case research while keeping an idiosyncratic focus through data replication (Wolery, 2013). This is usually achieved within studies using intra and inter-subject replication. However, it is also possible by changing the unit under analysis to small groups (Poncy, Skinner, & Axtell, 2010) or classrooms (Ervin, DuPaul, Kern, & Friman, 1998; Poncy, McCallum, & Schmitt, 2010).

Regarding academics, the behavioral analytic perspective also has greatly contributed to the empirical research base in screening/assessment and evidence-based intervention (EBIs). Behavior analytic philosophy is the basis of empirically supported techniques like Direct Instruction and Precision Teaching (Kameenui & Simmons, 1990; Lindsley, 1971). Behavior analytic assessment is also operated with the use of Curriculum-Based Assessment (CBA) and Measurement (CBM) tools (Deno & Mirkin,

1977). These assessments have greatly contributed to improved practice and toward the goals of accountability and improvement. Such measurement tools are considered dynamic indicators of basic skills in which academic behaviors are measured in discrete and objective terms using frequency dimensions that are sensitive to change (Shinn, 2002). That is, they are both specific enough to hold construct validity, but also sensitive enough to convey reliability of effect. Curriculum-Based Measurement holds additional value, because it is able to be standardized and used with explicit decision-making models (Deno, 1995).

Overall, while education and epistemology have been a focus in psychological research, increased accountability for schools has promoted the use of behavior analytic perspectives that are outcome-based to implicate practice. This is being achieved through several facets within assessment and intervention research. However, research designs that use direct measurement through CBM are considered a premium in the zeitgeist for evidence-based intervention research.

**Instructional Hierarchy**

A prominent behavior analytic model that is used in education is the Instructional Hierarchy (IH). Originally outlined by Haring and Eaton (1978), these authors propose an instructional hierarchy that matches the topography of student responding to specific instructional recommendations that range from building accuracy to complete mastery of a skill (i.e., adaption). Under this hierarchy, the first step towards mastery requires initial *acquisition* of a skill or behavior. Acquisition is followed by the need to be *fluent* in the behavior, in which responding can occur quickly and with ease when instruction is not provided. Once a skill meets these requirements, an individual must be able to *generalize* and *adapt* the behavior across new situations to be fully amalgamated into their

behavioral repertoire (Haring & Eaton, 1978). When educating through these sequential steps, a learner's instruction is functionally paired to their need.

**Building Accuracy**

The first stage of the IH, acquisition, is commonly the primary target observed in classrooms. However, this is only the first stage of skill mastery (Wolery, Bailey, & Sugai, 1988). Haring and Eaton (1978) defined skill acquisition as "the period between the first appearance of the desired behavior and the reasonably accurate performance of that behavior" (p.25). Behaviorally, skill acquisition is conceptualized as operant conditioning using a three-term contingency learning trial that includes an antecedent (i.e. discriminative stimulus), response, and a consequence (Albers & Greer, 1991). Other concepts of skill acquisition taken from basic research include stimulus control (Shahan & Chase, 2002). Because accuracy is defined as the quality of a corresponding response within the three-term contingency, it is best measured as number or percent of correct attempts and number or percent of errors.

When individuals fall into this stage, providing functionally related instruction is appropriate. Research has determined the use of explicit instruction (i.e. three-term contingency learning trials) results in the most effective strategy (Skinner, 1998). For instance, direct instruction consistently produces better outcomes than implicit methods of instruction (Gersten, Woodward, Darch, & Craig, 1986). Many additional components that reside within the three-term contingency have attributed to effective instruction for accurate responding. Antecedent conditions of demonstration, modeling, prompting/cueing, and errorless learning are effective techniques to increase accuracy (Carnine, Jones, & Dixon, 1994; Daly, Witt, Martens, & Dool, 1997; Hendrickson & Gable, 1981). Also, consequences directly tied to instruction also effectively improve

accuracy, including immediate feedback (Vargas & Vargas, 1991), error correction (Van Houten, 1984), and providing high rates of reinforcement (Daly, Witt, Martens, & Dool, 1997; Martens, Lochner, & Kelly, 1992).

**Building Fluency**

Once a skill can be performed accurately without assistance, an individual must become fluent (Haring & Eaton, 1978). Also referred to as automaticity in cognitive models (Hasslebring, Goin, & Bransford, 1987), fluency can be defined as the ability to perform a target skill at a rapid and proficient rate (Daly, Lentz, & Boyer, 1996). Because the topography of fluent responding is such that it depends on time, it is best measured as rate (i.e. frequency of behavior per unit of time).

Many educators frequently disregard this learning stage over accuracy building, but its inclusion into the IH is important for several reasons. First and foremost, reliance on the Instructional Hierarchy states that fluency is a prerequisite to develop generalized and adapted skills (Haring & Eaton, 1978; Skinner, 1998). From a practical perspective, those who are fluent with a skill are able to complete tasks at a faster rate. This allows learners to acquire (Albers & Greer, 1991) and retain (Ivarie, 1986) more information in less time. Increasing opportunities also increases academic endurance (Martens & Witt, 2004). Increasing fluency levels also increases rates of reinforcement (Skinner, 1998). Based on the Matching Law (Herrnstein, 1961), becoming fluent in an academic skill decreases response effort needed for the task, consequently increasing engagement (McDowell, 1988). Furthermore, being able to complete more behaviors in less time is reinforcing (Martens et al., 1992; Skinner, Fletcher, & Henington, 1996). Reinforcement from fluency is also important from a socio-cultural perspective by promoting health adjustment. Those whom are fluent in basic math hold less math anxiety (Cates &

Rhymer, 2003). Increasing fluency may promote academic competence and motivation, while demote peer rejection and antisocial behavior (Martens & Witt, 2004). Also, because fluent responding increases learning efficiency, educators need less time to revisit topics while allocating more time to discovery activities or social skills training (Skinner, 1998).

Many cognitive perspectives also promote the importance of fluency (i.e. automaticity) in developing academic achievement. Short-term working memory (Stankov, 1988) has been conceptualized as "an ability to hold information in the span of immediate apprehension while doing other cognitive things" (Horn & Blankson, 2005, p. 50). Because individuals' short-term working memory is limited, a lack of automatic responding may make one's ability attend to complete complex tasks, tasks with several steps, or multiple tasks simultaneously very difficult. Proper completion of such tasks may be easier if some of the steps involved require little cognitive effort to allow for more of one's cognitive attention on other steps. When applied to mathematics, becoming automatic in basic facts can allow one to allocate more cognitive resources to acquire or complete complex skills needed in math (Dehaene, 2011). For instance, a student who can respond to 3+5=8 "without thinking" will be able to put more conscious attention towards thinking about how to properly respond to 8-5= ___. This cognitive model is very accommodating to the IH where fluency is functionally related to generalization.

Opposed to the accuracy building stage that requires three-term contingency trials, building fluency is best achieved by "drill" and/or "practice" (Haring & Eaton, 1978). More specifically, fluency requires the quick and repeated engagement of two-term contingencies (prompt plus response) to increase rate of responding (Daly et al., 1996; Martens & Witt, 2004; Vargas & Vargas, 1991). Success in fluency building is

especially seen when such procedures promote high rates of responding (discussed further detail below) (Skinner, 1996).

**Promoting Generalization and Adaptation**

Once skills are accurate and can be performed fluently, hopefully learned skills can be utilized in situations outside of the instructional setting. That is, skills that can be successfully mastered in the instructional setting may be performed in either one's natural environment or a setting that differs from training (Haring & Eaton, 1978; Martens & Witt, 2004). In fact, independent of one's philosophic ideology, this definition of generalization is considered the true goal of education.

Generalization can be conceptualized along three dimensions, response, setting/situation, and time (Baer, Wolf, and Risley, 1968; Cooper, Heron, & Heward, 2007). Response generalization refers to Bear, Wolf, and Risley's (1968) "spreads to a wide variety of related behaviors" definition. That is, response generalization is seen when an individual can emit untrained responses that are functionally the same as intervened target behavior. For instance, a student who was trained to respond to the prompt "4+4" by verbalizing "8" and later is able to respond to the same prompt by writing the number 8 or holding up eight fingers would be expressing response generalization. This can also be conceptualized as teaching students addition with a set of facts with the hope that they will be able to appropriately respond to other addition problems that were not explicitly taught. This is important because educators cannot possibly begin to explicitly train every response that learners will need to engage in their lifetime.

Setting/Situation generalization refers to Baer, Wolf, and Risley's (1968) "appears in a wide variety of possible environments" definition. Setting/Situation

generalization is when an individual emits a target behavior in the presence of stimulus conditions (different settings, different people) that was not present in training. For instance, a student who was trained to accurately write a "6" to a prompt "3+3" written in a vertical format, but is then able to write a "6" for the prompt "3+3" written in a horizontal format would be expressing setting/situation generalization. This can also be conceptualized as teaching students addition with certain prompting tools (or strategies) with the hope that they will be able to appropriately respond to these same problems later with the absence of such prompts. This is important because educators cannot train all behaviors across all possible stimulus environments. Therefore, setting/situation generalization makes interventions more efficient because one only needs to intervene in one setting or situation. This form of generalization is the true goal of educators who express the need for students to "understand" what they learn.

Finally, generalization can be seen across time. Referred to as response maintenance, this generalization is defined as behavior that is "durable over time" (Baer, Wolf, and Risley 1968). Specifically, response maintenance is the extent to which a learner continues to perform the target behavior after a portion or all of an intervention accounting for change has been terminated. In this way, response maintenance generalization is resistance to behavioral extinction. This is important because generalization across time allows for individuals to continue performing socially significant behavior without constant intervention that could be timely or costly. It is also a main goal of educators to have their students remember what they learn.

Historically, basic research in operant conditioning considered generalization as a passive phenomenon in organisms' behavior and therefore a hindrance towards the science. The focus of research was in stimulus discrimination. Stimulus discrimination

was considered a more active process, requiring explicit training for achievement. As such, stimulus generalization was considered the natural result of failing to program stimulus discrimination (Skinner, 1953). That is, generalization of behavior happened naturally from poor or loose training.

Generalization has since become a more socially significant phenomenon (Stokes & Baer, 1977). This is especially pertinent in education, where it is hoped that students' learned behavior will not be constrained to the specifics of their learning environment. In fact, this new perspective on generalization directly influenced the construction of the IH as a vehicle for promoting it. Taken from basic research in stimulus gradients, Haring and Eaton (1978) identified two types of "practice" that take essential roles in regards to generalization. These included discrimination training and differentiation. Discrimination training can be defined as a procedure in which responses are reinforced in the presence of one stimulus condition, but not reinforced in the presence of another stimulus (Cooper et al., 2007). Alternatively, differentiation is a systematic form of stimulus generalization in which responses are reinforced to stimuli that are systematically different from the original stimulus under control (Cooper et al., 2007).

Stokes and Baer (1977) were among the first to present specific technologies to program generalization for socially significant human behaviors. In their classic article, they broke down discrimination training and differentiation into nine techniques to promote generalized behavior: (1) train and hope, (2) sequential modification, (3) introduce to natural maintaining contingencies, (4) train sufficient exemplars, (5) train loosely, (6) use indiscriminable contingencies, (7) train to generalize, (8) mediate generalization, and (9) program common stimuli.

*Sequential modification* is the process of systematically initiating training procedures in "every nongeneralized condition" (p. 352). That is, this technique is merely the application of procedures to produce accuracy in any/all settings where it is hoped the behavior will be engaged. One can prevent having to provide training in such a wide array of contexts by *introducing one to natural maintaining contingencies*. This is a technique in which trained behaviors will likely be exposed to natural contingencies to maintain the behavior in the environment in which generalization is hoped to occur. For instance, providing social skill training may be better generalized if the new environment contains friendly individuals that will reinforce interactive behavior (Stokes, Fowler, & Baer, 1978).

*Training sufficient exemplars* is similar to sequential modification by incorporating training sessions across different stimuli conditions, but does not utilize training in all nongeneralized conditions. Alternatively, it is hoped that one can provide a limited yet *sufficient* amount of exemplars across settings, trainer, stimulus, or response topography during practice. This allows for consistency to ensure stimulus control while also incorporating variability for generalization. *Training loosely* follows the same logic by using variability within training. Best defined as "the negation of discrimination" training (Stokes & Baer, 1977, p. 357), this somewhat informal technique merely hopes to achieve the direct opposite of discrimination by applying the opposite training principles.

*Using indiscriminable contingencies* intends to promote generalization by providing conditions that disallow an individual to predict if emitting a learned response will or will not be reinforced in certain situations. Intermittent schedules of reinforcement are less predictable and therefore more resistant to extinction than continuous schedules

(Ferster & Skinner, 1957). Using indiscriminable contingencies uses this same logic by making it particularly useful to promote response maintenance (Stokes & Baer, 1977).

*Mediating generalization* and *training to generalize* may hold the least practicality. First, mediating generalization is a procedure defined by Stokes and Baer (1977) that "establishes a response as part of the new learning that is likely to be utilized in other problems as well, and will constitute sufficient commonality between the original learning and the new problem" (p. 361). This technique is included primarily to offer proposals for generalizing verbal behavior, self-control, and self-managing procedures, but the authors acknowledge this method as highly theoretical. Secondly, if generalization is merely a form of response, then placing reinforcement contingencies upon the behavior should result in skill development as with any other operant condition. Although this method frequently fails, training to generalize also includes the systematic use of instructions, which holds more merit (Stokes & Baer, 1977).

*Train and hope* and *programming common stimuli* work from the assumption that "after a behavior change is effected through manipulation of some response consequence, any existent generalization across responses, settings, experimenters, and time, is concurrently and/or subsequently documented or noted" (Stokes & Baer, 1977, p. 351). The train and hope method takes this assumption to wait for generalization spontaneously occur once enough stimulus combinations have taken place during training. The technique of programming common stimuli exploits this assumption by actively manipulating certain common, salient, stimuli to be present across both the trained and generalization settings. This facilitates generalization past the train and hope method by providing functionally relevant associations and excluding interference. For instance, Stokes and Baer (1976) taught two children with learning disabilities word-recognition

skills, one of the children acting as a peer tutor for the other during teaching. While both children learned the skill in the trained setting, generalization was not seen to a different setting. However, generalization from both children was seen in the generalized setting only when the students were instructed to work in the peer-tutoring situation was incorporated. Similarly, Walker and Buckley (1972) programmed generalization of social and academic behavior by using the same (common stimuli) academic materials across environments.

**Difficulties in Generalizing**

The first, and only implicit, of these technologies from Stokes and Baer (1977) (train and hope) is not a form of discrimination training or differentiation. However, it is directly reflected within the first two stages of the instructional hierarchy. By merely teaching accurate responding and providing sufficient amounts of drill to build fluency, generalization would be the next natural step in one's skill development (Haring & Eaton, 1978). This makes intuitive sense because being fluent in a behavior allows for increased responding in that behavior, which in turns allows for more opportunities in various discrimination and generalization learning trials (Skinner, 1996). In fact, the majority of empirical literature that has produced generalized academic behavior utilizes this "train and hope" sequence without active programming (discussed further below) (Codding & Poncy, 2010).

These findings emphasize the significance of high fluency rates for generalized responding. In their original work, Haring and Eaton (1978) communicated the importance of drill on fluency and proficiency to facilitate retention (i.e. response maintenance), as well as allowing behavior to be socially significant throughout life (i.e.,

response and setting/situation generalization). However, they also state that specific instructional tactics are probably required for generalization in most instances.

Stokes and Baer's (1977) technology to program common stimuli share the same underlying assumptions of the train and hope phenomenon, thus fitting well within the IH framework. Being fluent in a behavior allows for increased responding in that behavior, in turn allowing for more opportunities in discrimination and generalization learning trials (Skinner, 1996).

While a necessary function, Haring and Eaton (1978) further state that fluency is not the sole function of generalization. Alternatively, they recommend the use of "practice" to foster skill generalization. While drill can be defined as "the directed repetition of the specific response to be learned," practice includes "utilizing the learned response *with other learned responses* to solve a variety of problems" (p. 30). Based on the present literature on the IH, one may infer that generalization procedures like programming common stimuli would be inappropriate to incorporate into acquisition and fluency building, and should be statically placed in the generalization stage of skill development. For example, attempting to promote generalization within the accuracy phase may be counter intuitive as a direct contrast to discrimination training. Similarly, incorporating additional stimulus prompts and/or requiring more complex responding during the fluency stage may impede the speed of response rates of two-term contingencies. However, the original architects of IH indicate flexibility when it comes to generalization training as they state:

> Clearly, it is quite probable that different instructional tactics would be required to teach generalization of skills than those tactics appropriate at other stages of learning. The point at which a child is ready to begin this

> phase of learning, however, is unknown. It may be, for example, that procedures designed to enhance cross-situation generalization should begin concurrently with procedures designed to enhance fluency and maintenance (Haring & Eaton, 1978, p. 30).

Being fluent no doubt makes complex generalization programming easier. Yet effective approaches of programming common stimuli "encourage the incorporation into training settings of (naturally occurring) physical stimuli that are frequently prominent or functional in non-training environments" (Stokes & Baer, p. 360). That is, the best use of programming common stimuli would plan ahead to include stimuli relevant to generalization in accuracy or fluency stages, especially if it is possible to avoid techniques competing with discrimination training and/or learning rate. As this suggests, "practice" in the form of programming common stimuli may be engaged during the fluency stage as well.

Practice could also result in more efficient generalization if the common stimuli incorporated are selected and trained appropriately. Specifically, Stokes and Baer (1977) state "one predictor of the salience of a stimulus to be chosen for this role is its already established function for other important behaviors of the subject" (p. 360). That is, stimuli that are shared across conditions are most powerful if they are directly required to engage in both of the behaviors. In a sense, including salient stimuli for these reasons could increase learning trial strength (Skinner, 1998).

**Learning Effectiveness vs. Learning Efficiency**

The concept of learning is difficult to operationally define. Being considered by some as an internal mechanism (Dubinsky & Mcdonald, 2002; Larochelle et al., 1998; Piaget, 1952), behavioral definitions define learning as permanent change in behavior

that is brought about by experience (Skinner, 2008). To add, learning is best conceptualized as a rate at which people change their behavioral repertoire across time. Therefore, learning problems are not caused by a failure to learn, but a failure to learn fast enough (Skinner et al., 1996). This perspective on learning iterates the dualistic nature of academic problems and suggests greater focus be put on feasibility and resource-efficiency when evaluating the utility of interventions. In fact, the original development of the IH was initiated in response to the lack of attention put towards efficient learning. For instance, teaching new skills in sequential fashion using three term contingencies (prompts and consequences to teach behavior) would result in more academic learning than implicit methods when controlling for time (Haring & Eaton, 1978).

Although cumulative instructional time is considered important when evaluating instruction, it still may be overlooked in supplemental interventions (Barnett, Daly, Jones, & Lentz, 2004). A meta-analysis by Bramlett, Cates, Savina, and Lauinger (2005) found that from the ten publications evaluating mathematic interventions up to 2005, only two took into account instructional time. Of these studies, both indicated that decisions regarding an intervention's relative effectiveness can be impacted by factoring in instructional time on a session-by-session basis (Cates et al., 2003; Watson & Ray, 1997).

Research evaluating intervention efficiency is limited, but methods to account for this facet have been identified. Learning rate (i.e., amount of items learned per instructional min) has been used as a valid measure of intervention efficiency and has been incorporated into the practice of evaluating academic interventions (Joseph & Nist, 2006; Joesph & Schisler, 2007; Schmidgall & Joseph, 2007; Skinner, Belfiore, Mace, Williams-Wilson, & Johns, 1997).

Many explicit interventions have been provided in the literature that accomplishes time-efficient remediation (Burns, Codding, Boice, & Lukito, 2010). However, it is equally important to identify what behavioral mechanisms and components contribute to increased learning rates when utilized within such instructional procedures. Two main mechanisms of efficient interventions are increased strength of responding and increased rate of responding (Skinner et al., 1996). Increasing response strength allows for more learning to occur per response. Alternatively, increasing response rates allows for more responding per unit of time. Increasing response rate and response strength is accomplished using a variety of components. For instance, response rates can be increased by setting time limits (Derr & Shapiro, 1989) or limiting breaks and increasing pace (Darch & Gersten, 1985). Response rates are also increased when using more efficient response topographies (Gardner, Heward, & Grossi, 1994; Skinner, Ford, & Yunker, 1991), such as using group/choral responding (Sindelar, Bursuck, & Halle, 1986). In turn, increasing the saliency antecedent stimuli increases response strength (Van Houten & Rolider, 1990). Response strength is also promoted with consequence strategies such as providing feedback, goal setting/self-monitoring, and delivering positive reinforcement (Van Houten, Hill, & Parsons, 1975).

While the above components have been empirically validated to increase learning rate, studies directly comparing treatments have primarily taken a univariate approach by examining single dependent variables. That is, learning rate has mainly been evaluated using a one-dimensional effect on increasing a single, discrete unit across time. Taking a univariate approach prevents the evaluation of response generalization of interventions by disregarding other variables related to learning outcomes (e.g., generalized effects), possibly masking true effects. Therefore, a more *comprehensive* (i.e., multivariate) take

on instructional time should be seriously considered as an important outcome metric. Cronbach and Snow (1977) state:

> Learning is multivariate, however. Within any one task a person's performance at a point in time can be represented by a set of scores describing aspects of the performance. Hence, typical discussions of "learning rate" simplify drastically. They lead the investigator to collect a single score at each point, and so limit him to asking simple questions. (p. 116).

Examining intervention outcomes in this way is very applicable to the field of education and academic deficits where time and resources are at a premium. Conceptualizing multivariate measurement to assess generalization outcomes is useful because "any worthwhile treatment will affect the subjects in more than one way" (Stevens, 2009, p. 2). It is also likely that simultaneously targeting multiple skills is less costly than implementing multiple treatments (Stevens, 2009).

This is seen more clearly in a hypothetical example applied to the present literature. Poncy, Skinner, and Jaspers (2007) compared the interventions Cover, Copy, Compare (CCC) and Taped Problems (TP) in increasing two problem sets (one assigned to each intervention) of basic addition. Although CCC showed a higher fluency level after seven sessions (30 digits correct per minute (DCPM)) than TP (25 DCPM), the length of sessions for each intervention was not equal. After comparing interventions based on learning rate, it was determined that the growth rate of CCC (.54 DCPM increase per instructional minute) was less than that of TP (.76 DCPM increase per instructional minute). These results iterate the value of learning rate as an outcome metric beyond that of learning level. Yet hypothetically speaking, what if the reciprocals of each

problem set were simultaneously measured? Furthermore, what if CCC's use of salient modeling and feedback was able to increase both the targeted problem set as well as their reciprocals at a rate of .54 DCPM per instructional time, while TP did not promote generalization to the reciprocals? Examining both growth rate and generalization effects in this way would again turn the tables by determining that CCC growth rate of 1.08 DCPM (.54 of basic addition and .54 of reciprocals) promotes a higher *comprehensive* learning rate than that of TP (.76 of basic addition and .00 of reciprocals).

Cronbach and Snow (1977) stated that as "Multivariate studies of the course of learning being almost nonexistent, no review of findings can be undertaken" (p.116). Unfortunately, little has changed since their discussion in 1977 and the lack of research in academic generalization still disallows us to predict how known interventions (and intervention components) contribute to promoting generalization in an efficient manner. For instance, because it is generally observed that increasing trial rate is most efficient for increasing fluency, it may be that this same method promotes generalization in the same manner. On the other hand, it may be that treatment practices that increase trial strength (e.g., technologies for generalization (Stokes & Baer, 1977)) may decrease trial rate, but may be more promising for efficiently promoting *comprehensive* learning rates (i.e., generalization) (Haring & Eaton, 1978). Therefore, it may be suggested that similar focus on learning rate be put on generalization research.

**Universal Class-wide Interventions**

Raised awareness to focus on learning rate over learning has allowed the role of the school psychologist to expand. Traditionally intervention services have often been designed to help one child at a time. In an effort to increase the efficiency of service delivery models educators have begun to focus on "big problems" (Shapiro, 2000). Such

interventions are delivered in an identical manner and under identical conditions to all targeted students (Gresham, 2004). When initiated, it is estimated that such interventions are effective with approximately 80-90% of students (Sugai, Horner, & Gresham, 2002). This increases instructional efficiency for many reasons. First, instead of spending time with individuals, it is more time and resource efficient to apply evidence-based interventions universally to serve many individuals at once (Hawkins, 2010; Rathvon, 1999). Second, delivering effective treatments at a group level saves time later that would have been devoted to remediation (Skinner, 2008). Third, providing class-wide intervention ensures that all students are engaged, rather than just the students referred for intervention (Elliott, Turco, & Gresham, 1987).

Intervening on a universal level is important for many reasons beyond instructional efficiency. First, the majority of research in behavioral analytic academic interventions target special education students that have skill deficits. Therefore, the focus of much of the research is validating treatments to remediate rather than prevent skill deficits (Skinner, 1998). Yet it is very likely that procedures shown to remediate skills would also prevent skill deficits in the general population. Students who are referred due to a lack of response to intervention (low learning rates) are more likely to be in true need of more intense instruction (e.g., alternate placement). Therefore, intervening at the universal level may lead to more accurate decisions regarding special education placement (Gresham, 2004). Using evidence-based practices class-wide is conducive to the perspectives of educators taking constructivist approaches that focus on personal domains related to academics. Specifically, remediating students' academics skill deficits may promote students' academic competence and resilience (Shapiro, 2000) as well as decrease academically-based anxiety (Cates & Rhymer, 2003). Also, targeting

academics may indirectly decrease behavioral problems by promoting academic engagement (Rathvon, 1999). Finally, many evidence-based interventions require technical training and resources that may not be available in the general classroom. Therefore, further research should target useful applications of EBIs in the general education setting (Skinner, 1998).

A number of studies have examined evidence-based academic interventions at the group level (Aspiranti, Skinner, McCleary, & Cihak, 2011; Axtell, McCallum, Bell, & Poncy, 2009; Codding, Chan-Iannetta, Palmer, & Lukito, 2009b; Codding et al., 2007b; Poncy & Skinner, 2011; Poncy, Skinner, & McCallum, 2012; Poncy, Skinner, & O'Mara, 2006; Rhymer et al., 2002) . Positive results have been found over a variety of subject areas, including early literacy skills (Gajus & Barnett, 2010). A number of studies have also evaluated evidence-based treatments in mathematics at the class-wide level. Of the studies evaluating mathematics, the majority targeted math fact fluency as the dependent variable. Only one study (Codding et al., 2009b) observed generalization at the group level.

Although this research consistently shows that evidence-based interventions can successfully be applied at the group level, there are still few studies analyzing evidence-based academic interventions (Gresham, 2004). While the research mostly targets fluency building, the present research also rarely measures their effects while controlling for instructional time (Skinner, 2008). Finally, few studies have observed group effects regarding generalization. Overall, evidence-based intervention research has provided much insight in skill remediation, but the gaps in class-wide analysis is a disservice to general educators also in need of evidence-based techniques. Therefore, these variables should be the focus of future research.

**Evidence-Based Interventions for Mathematics**

Using the Instructional Hierarchy as a template, the fields of Education and School Psychology have achieved great strides in identifying evidence-based practice to promote students' learning of basic math facts. Two prominent interventions found in the literature that target these academic skill deficits are Cover Copy Compare and Explicit Timing.

Cover Copy Compare (CCC) is an evidence-based intervention that targets both accurate and fluent responding of academic stimuli. This procedure involves a student observing an academic model, covering the model, responding, and checking one's answer with the model (Skinner et al., 1997). Through these simple steps, CCC is a self-managed intervention composed of complete and repeated learning trials. As such, this intervention includes many effective behavioral-change components such as modeling, immediate feedback, error correction, reinforcement, and exposure to repeated learning trials (Skinner et al., 1997).

While originally developed as a spelling intervention (Hanson, 1978), CCC has been applied to a variety of academic stimuli including science (Smith, Dittmer, & Skinner, 2002) , geography (Skinner, Belfiore, & Pierce, 1992), number writing (Saecker, Skinner, Sager-Brown, & Roberts, 2009; Skinner, Saecker, Hawthorn, & Sager-Brown, 2009) and mathematics (Poncy et al., 2007; Poncy et al., 2012; Skinner et al., 1997; Skinner et al., 1989). In the area of mathematics, the basic steps have include the following: (1) present the student with worksheets containing completed academic models (i.e. the math problem and correct answer), (2) have the student observe the model, (3) have the student cover the model (i.e. cover the complete problem with non-writing hand), (4) have the student write the problem and answer in the absence of model

from view, and (5) have the student again review the completed model and compare it with their own response (Skinner et al., 1997). Using this template, many studies have directly observed the effectiveness of Cover Copy Compare to remediate math-based deficits (Joseph et al., 2012).

Skinner et al. (1989) were the first to observe the experimental effects of Cover Copy Compare regarding mathematics. Using a within single-subject multiple baseline design, this study evaluated CCC's effect of increasing multiplication accuracy and fluency rates of four students. Students were engaged in CCC procedures for three minutes per day. Results indicated that the intervention effectively increased both accuracy and response rate for all subjects across seven days of intervention when compared to baseline levels. These results indicate CCC as time and resource-efficient intervention for math accuracy and fluency.

Similar studies have also found CCC to be an effective intervention for remediating math performance across a variety of populations. For instance, CCC can effectively increase math performance for learning disabled populations (Stading, Williams, & McLaughlin, 1996), students with intellectual disabilities (Poncy, et al., 2007) as well as students without disabilities (Grafman & Cates, 2010; Lee & Tingstrom, 1994). Cover Copy Compare has also been shown be effective with students diagnosed with behavior disorders (Skinner et al., 1989). Studies have also confirmed the efficacy of CCC across race and ethnicity (Joseph et al., 2012).

Cover Copy Compare has also been shown to be effective when using additional response topographies and additional instructional components. Skinner, Ford, and Yunker (1991) compared two forms of CCC, one requiring verbal responses only and the other requiring the traditional writing response using an alternating treatment design.

While controlling for intervention duration, changing the topography of responses to verbal resulted in more than twice the response rate within session, as well as greater increases in written performance than the traditional written treatment (Skinner et al., 1991). Similarly, another study used an alternating treatment design to compare CCC with peer-delivered corrective feedback and self-delivered corrective feedback on multiplication performance (Skinner, Shapiro, Turco, & Cole, 1992).

The same author utilized an across-task multiple-baseline design to evaluate a "Cognitive" Cover Copy Compare intervention for division digits correct per minute (DCPM) in three students (Skinner et al., 1993). This CCC procedure had students subvocalize the math problem model rather than the traditional vocalization of the model. For two of the three students, CCC using subvocaliztion resulted in increased rates of division responding compared to baseline levels, while the third increased with the addition of feedback and goal-setting. Further research has also found that incorporating goal setting within CCC increases growth rates over CCC without goal setting (Codding et al., 2009b). A study by Poncy and Skinner (2011) also found that incorporating components of timed practice and rewards to CCC procedures enhances results when self-administered. Codding et al. (2007a) also found that using CCC while giving performance feedback on digits correct and incorrect increase fluency rates, but not more than CCC alone. The entirety of these results support CCC as an empirically-supported self-administered intervention and presents recommendations to use behavioral components to promote learning rates.

While the results above indicate CCC is superior to baseline phases containing no instruction, several studies have directly compared CCC against alternative treatment methods. A study by Poncy, McCallum, and Schmitt (2010) compared CCC with a

constructivist technique, an approach that is popular with many educators. Through an alternating treatments design, CCC using subtraction fact triangles was compared with a constructivist intervention taken from "Facts that Last" (Leutzinger, 2002a). After 10 sessions of each treatment, CCC was found to effectively increase digits correct per minute of simple subtraction facts, while the constructivist approach did not differ from controls. Not only did this study present the effectiveness of CCC over constructivist approaches, but indicated that the use of the fact triangle as a model using CCC procedures is an effective intervention technique.

Poncy et al. (2007) compared CCC with the intervention Taped Problems (TP). Taped Problems (McCallum, Skinner, & Hutchins, 2004) targets both accurate and fluent responding by pairing audio cues with a worksheet containing computation problems. Specifically, an audio recording is started and reads each problem, followed by a brief delay (1-4 seconds), and then the answer. Students are instructed to follow along and try to beat the tape by writing in the answer before the recording provides the correct response. The recording paces students across a series of problems incorporating high rates of responding and immediate feedback. Using an alternating treatments design, the present study found that both CCC and TP equally increased the accuracy and fluency rate of basic addition in an intellectually disabled student. However, because the Taped Problem procedures took approximately 30% less time to complete, this study indicated that the TP intervention was more time-efficient.

Through an alternating treatments design, Mong and Mong (2010) compared CCC with Math to Mastery (MTM; Doggett, Henington, & Johnson-Gros, 2006). Math to Mastery is an explicit instruction approach that has the student (1) complete each math problem with an instructor, (2) completes each math problem within a 1-minute trial

while the instructor simultaneously provides immediate formative and summative feedback, and (3) and completes a self-monitoring chart. While both interventions were successful at increasing DCPM of mixed simple addition and subtraction skills, the researchers found that the Math to Mastery intervention was more effective in two of the three participants. Specifically, MTM scores for the two students were only 3 and 5 DCPM higher than CCC, but MTM sessions took twice as long as CCC sessions (10 min versus 5 min). Therefore, MTM was caused more learning, but CCC resulted in a higher learning rate (Mong & Mong, 2010).

Although CCC has been empirically shown to increase accurate and fluent responding, a second evidence-based intervention used to enhance academic fluency is explicit-timing (ET). As stated above, building fluent responding can best be achieved through high rates of drill and practice (Daly et al., 1996). Fluency gains can be further increased by providing contingent reinforcement for increasing academic output rate (Chadwick & Day, 1971). While this practice of providing fixed-time contingencies (i.e. fixed-interval schedule of reinforcement) was valued among educators in early practice, there were inconsistent findings in its effectiveness (Semb & Semb, 1975). However, early practices of timed-assignments did not communicate the timing expectations other than when to begin and end working. That is, they were not explicitly timed (Van Houten, Morrison, Jarvis, & McDonald, 1974).

A study by Van Houten and Thompson (1976) was the first to present effective findings for explicitly timed practice in mathematics. They compared baseline (no timing contingency) condition to an ET condition. Individuals were explicitly timed and were told that they had one min to complete as many math facts as possible. Within an ABAB single-subject design, students increased their rate of problem completion from 6.8

correct problems per minute (1st baseline) to 10.5 correct problems per minute (first treatment phase). Upon return to baseline, the students decreased their responding to 5.5 correct problems per minute (2nd baseline), but again increased to 11.5 correct problems per minute (2nd treatment phase).

Miller, Hall, & Heward (1995) extended the Van Houten and Thompson (1976) study by directly comparing seven 1-minute explicit timing conditions to an implicit timing conditions in which student were instructed to "work fast" for 10 min. Because Van Houten and Thompson's (1976) baseline condition had no teacher instruction, implicitly instructing students to "work fast" reflected a more accurate educational environment. They also evaluated an additional condition in which students received only two 1-min timings but received teacher feedback and self-correction. Using an ABABCBC single-subject design applied to a class of first-grade regular and special education students, they determined that explicit timing was superior to more implicit timing activities in both studies. Their study also indicated that both the first-grade classroom and special education classroom emitted faster responding under the ET condition than the condition that included teacher feedback and self-correction (Miller et al., 1995). However, because the ET condition without teacher feedback and self-correction engaged in more practice trials, it is undetermined which condition would improve more over time. Nonetheless, this study indicates superiority in not only ET but also recommends the addition of feedback and self-correction procedures.

Several studies show that ET is not only effective in raising math response rates within session, but increases DCPM scores over time. For instance, ET has been shown to increase fluency across races and ethnicity groups (Evans-Hampton et al., 2002; Rhymer et al., 1999; Rhymer et al., 1998), as well as those with average and below

average intellectual functioning (Miller et al., 1995). Explicit Timing has also shown to be effective across mathematics skill difficulty (Rhymer et al., 2002).

A prevalent attitude for many educators is the value of accuracy over fluency (Cooke, Heward, Test, Spooner, & Courson, 1991). Similar concerns have been cited that ET may make students "work too fast" in which they are more prone to make mistakes and be inaccurate (Van Houten, 1984). These concerns have not been validated. Only one study (Rhymer et al., 1998) observed this effect, but it was not caused by the ET procedure nor did it occur in subjects who attained accurate levels pre-intervention. Yet, ET has been considered evidence-based for increasing fluency only (Codding et al., 2007b; Miller et al., 1995). Therefore, as Rhymer et al. (1998) state: "Timing procedures to increase rates of active responding should either ensure that students have obtained sufficient accuracy levels before implementing the timing procedures or supplement timing procedures with procedures that are designed to increase response accuracy" (p. 676). Such a procedure could be CCC.

**Intervention Comparison**

Both CCC and ET possess a robust research base that distinguishes them as evidence-based interventions. However, as noted above, it is also important to differentiate between interventions to identify those that result in learning gains but also high learning rates (i.e., learning per instructional min). Only one article has analyzed their efficacy in regards to each other (Codding et al., 2007b). In this study, Codding et al. (2007b) examined the difference in fluency growth between CCC, ET, and a control condition. They also took specific examination of skill-level by intervention interaction in improving fluency rates measured in digits correct per minute (DCPM) and digits incorrect per minute (DIPM). Through a randomized group experimental design, 98

second and third grade students were assigned to one of the three conditions (i.e. CCC, ET, control). The two experimental groups engaged in the treatments for five minutes twice weekly over a six week time period.

Their results, employing hierarchical linear modeling, indicated that while there was a significant main effect for improvement over time in DCPM. There was also a significant simple main effect for initial fluency level, in which high initial fluency significantly predicts increased performance over time. However, there were no differences between the three groups at this level of analysis. There was also a significant interaction effect between group membership and initial fluency level on DCPM. That is, those in the ET group that had low fluency levels initially ($\leq$10 DCPM) improved the least of all groups in performance over time while those with high fluency levels initially ($\geq$10 DCPM) improved the most of all groups in performance over time. No such interaction was found within the CCC or control groups. Regarding digits incorrect per minute (DIPM), individuals in the ET group decreased errors faster over time (improved performance in accuracy) over those in the CCC or control group. However, the authors explain this finding as an artifact of high variability in errors from session to session. Overall, the authors state that the "results suggested that initial level of fluency is an important factor in determining intervention effectiveness (Codding et al. 2007b, p. 613)."

Several assumptions can be inferred from these findings. Their results follow the rational of the IH which pairs instructional recommendation with student response topographies. Specifically, students who respond inaccurately need a three-term contingency while accurate responders will benefit from a two-term contingency promoting practice and reinforcement. Because ET utilizes two-term contingency

responding to allow for efficient responses, those able to take advantage (those with initially high levels of fluency) will benefit. Including modeling steps for those at this skill level may merely be adding white noise. Alternatively, those in the frustrational range may actually still be developing stimulus control and still be requiring three-term contingencies that incorporate modeling. These inferences have large implications to practice applying the instructional hierarchy in which instructional level dictates treatment

Many limitations of this research also necessitate skepticism. First, inferences from interactions are only applicable to ET. Predictive growth of CCC does not depend on initial fluency levels. Secondly, while statistically significant, the practical significance of this interaction should also be included when informing practice. CCC involves more steps that limit opportunities to respond, yet still showed a simple main effect demonstrating statistically significant growth over time. Explicit Timing growth (.58 DCPM/session) was found to be significantly higher than CCC growth (.38 DCPM/session) for those of initially high fluency when inferring longitudinal trajectories, yet the empirical differences in growth rate came out to .4 DCPM/sessions per week.

Finally, Codding et al. (2007b)'s results only identified possible skill by treatment interactions in regards to fluency building. Considering the majority of evidence-based intervention research uses this dependent measure because of its educational utility, this is not a limitation per se. Yet one cannot overextend findings beyond the dimensions under which they were found. For instance, ET may result in higher rates of fluency building than CCC under certain conditions, only to become erroneous when evaluating

generalization. This is one dimension that is vastly underutilized in such research, and should be incorporated into evidence-based intervention research.

**Interventions for Generalization**

As stated previously, Stokes and Baer (1977) identified nine methods to actively program behavior generalization. While the IH assumes accuracy and fluency as prerequisites for generalization (Ardoin & Daly, 2007; Binder, 1996), there is a lack of research validating the other active technologies in promoting generalization in academics. Training sufficient exemplars and programming common stimuli have been the only methods found to be successful in the academic literature.

A study by Silber and Martens (2010) compared multiple exemplar training (ME) of target words within reading passages with a regular listening passage preview repeated reading (LPP/RR) intervention and a control group. The results found that those receiving ME intervention (representative sentences containing target words) did not differ in learning rates from the LPP/RR group on passages intervened on, but showed greater learning increases on a generalized probe not intervened on.

Duhon, House, Poncy, Hastings, and McClurg (2010) used a multiple baseline design on three students to evaluate the effects of fluency building, cueing strategies, and exemplar training on letter sounds on the generalization towards sound blending. While they found that fluency building alone worked in one individual (train and hope), the cueing strategy (program common stimuli) as well as exemplar training worked in the other respective students. Programming common stimuli has also been found successful in generalizing known words to unknown words with similar sound structure (Mesmer, Duhon, & Dodson, 2007).

The train and hope method is the only method identified in the literature applied to mathematics. Regarding the generalization of mathematics, accurate and fluent responding are assumingly essential before generalization into more advanced skills can occur (Binder, 1996). Research has shown strong correlations between fluency and generalization of related skills. Van Der Heyden and Burns (2008) found that fluency scores in basic math skills are positive predictors in later performance in related complex skills. Similar research has also found that students performing basic math skills fluently could significantly predict subsequent learning rate of related math skills (Van Der Heyden & Burns, 2009). Strong correlations as high as 0.74 has been shown between fluency scores in single-digit multiplication and multiple-digit multiplication fluency (Lin & Kubina, 2005).

Other research has found functional relationships between fluency building and generalization using the train and hope method. A study by Codding, Eckert, Fanning, Shiyko, and Solomon (2007a) compared the effect of CCC with and without performance feedback had on generalization. Within an alternating treatments small-N design, individual students engaged in both CCC with and without regrouping utilizing either single-digit multiplication (two students) or three-by-two addition with regrouping strategies. Those receiving intervention in single-digit multiplication were assessed but not trained in three-by-one multiplication with regrouping to observe generalization effects. Those being intervened in three-by-two addition with regrouping were assessed but not trained in three-by-three addition with regrouping. While their results showed that all students increased fluency in the target skills across both treatments, there were only slight increases to the generalized skills.

Codding et al., (2009a) used brief experimental analysis on four students to observe generalization of fluent responding across untrained problems of similar difficulty. Using generalization probes consisting of half trained and half untrained items, one student was able to increase fluency scores across probes. However, three of the four students were unable to do so.

While the studies above show mixed results regarding math generalization, some studies have observed meaningful outcomes. Codding et al. (2009b) compared CCC with and without goal setting (errors and correct digits) on single-digit subtraction fluency in a group context. They also used pre and posttest assessments of AIMSweb third-grade general outcome measure (GOM) probes and two-by-one subtraction probes to observe generalization between groups. All groups improved their fluency rates over time, but the CCC with goal setting on correct digits (CCC+GSC) improved the most. Analysis of covariance (ANCOVA), with pre-test scores and class used as covariates, along with Bonferroni adjustment for multiple comparison post-hoc follow up were used to observe generalization. Results showed that the CCC+GSC group had significantly higher post-test scores on the GOM and two-by-one subtraction probes than the control and CCC with goal setting on errors group. That is, those receiving CCC+GSC were able to generalize their improvement from single-digit subtraction to two-by-one subtraction and to a variety of mathematical skills of similar difficulty. Because this group also had the highest fluency rates in the target skill, it is undetermined if fluency or goal setting contributed to generalization.

Codding, Archer, and Connell (2010) observed the effect of incremental rehearsal on single-digit multiplication to successfully generalize skills of similar and more complex difficulty. Using a multiple probe design across multiplication probe sets, the

researchers assessed the ability of fluent responding on fact sets to generalize fluency scores on single-skill mastery probes (SSM), multiplying fractions, and word problems. Their results indicated that increased rates of single-digit multiplication problems successfully generalized to both single-skill mastery probes and fraction multiplication, and slightly for word problems.

Initially in their intervention, there was an inverse relationship between percentage correct during incremental rehearsal and DCPM. The authors attribute this to a transition between accurate and fluent performance. That is, their participant had difficulties differentiating between known and unknown stimuli before necessary practice. Generalization to SSM only occurred once known problems could be accurately discriminated from unknowns and could be applied to SSM probes. Therefore, they propose that while drill is important to increase response rate, composite practice may also be required to yield generalization (Haring & Eaton, 1978).

Results from the studies above indicate the utility of fluency building under the train and hope method to promote generalized responding. Yet because of mixed results, it may also be determined that fluent responding is necessary but not sufficient to actively promote generalization. Regarding these gaps in the research some hypotheses can be drawn. When generalization was seen in the three Codding articles above, the topography of target and generalization behaviors were very similar. For instance, consistent generalization has been seen between trained math facts and untrained math facts of the same skill or with general outcome measures containing trained items (i.e. single digit subtraction to GOM containing single-skill subtraction, SSM) (Codding et al., 2009a; Codding et al., 2009b; Codding et al., 2010). Generalization was also seen when target and generalized skills comprised of the same base operation but only slight changes in

needed procedures (i.e. single-digit subtraction to two-by-one subtraction) (Codding et al., 2009b). Alternatively, generalization was not programmed from fluency alone when target and generalized behaviors were less similar (i.e. requiring regrouping, word problems, different operation) (Codding et al., 2007a; 2010; Poncy, et al., 2010). Therefore, the more similar stimuli are, the more likely generalization will occur (Codding et al., 2010; Poncy et al., 2010).

Poncy et al. (2010) attempted to go beyond fact fluency alone to promote generalization. These authors hypothesized that students at fluent levels would be able to overcome previous barriers if taught explicit strategies to apply already fluent skills. That is, targeted skills will only be generalized to dissimilar skills if the additional procedures are explicitly taught. Using a multiple baseline design, these authors investigated the effects of fluency building plus instruction and practice of a conceptual procedure to program common stimuli in promoting generalized responding of addition facts to related subtractions facts. The first strategic procedure was "think addition" (Leutzinger, 2002a), a strategy that hopes to utilize fact families shared between addition and subtraction. Next, a cloze procedure related to the conceptual lesson was reapplied to addition practice. Consistent with previous research, fluent responding in addition facts was insufficient in generalizing to a highly dissimilar skill of subtraction. However, fluent students were also unable to generalize even after practicing the procedure. Therefore, it was determined that fluent levels applied to strategies are still insufficient for promoting generalization.

Although no generalization was found from their results, Poncy et al. (2010) discussed intriguing deductions for future research. In their study, the researchers followed the IH in which they programmed for generalization once declarative skills

were fluent. However, studies examining generalization of increasingly complex skills have involved the combination of fluent declarative skills with non-fluent procedural skills (Codding et al., 2007a; Poncy et al., 2010). In these situations, fluent responding in the generalized skill is restricted to the fluency rate of slow procedures (Bender, 1996). This dualistic concept reflects the need for "practice" as well as "drill" in promoting generalization (Codding et al., 2010; Haring & Eaton, 1978).

Regarding the inclusion of procedural knowledge into the IH, lack of generalization in previous research necessitates a need to reevaluate how we compare the topography between target and generalized behaviors (Poncy et al., 2010). While this relationship is not foreign in the literature (Codding et al., 2010), it needs to be further operationalized. Poncy et al. (2010) state:

> The further away the target behavior for generalization is from the original trained behavior, the less likely it will be to show the generalization of fluent behavior and the more likely that procedural skills will be needed to link the two behaviors (p.86).

This assumption makes intuitive sense when evaluating past research. Codding (2009b) found generalization from trained problems to untrained problems of the same operation. Because no additional procedures were needed to perform the generalized behavior, fluent responding of the original declarative skills was uninhibited under the new stimuli. This study also found generalization from single-skill subtraction to two-by-one subtraction. While slightly different topographically, very few new procedures needed to be incorporated and easy transition between responses occurred. As in Stokes and Baer (1977)'s technique of programming stimuli, highly common settings require less programming of common salient stimulus associations. If multi-digit subtraction

with regrouping was the generalized behavior, the incorporation of column alignment and regrouping strategies would likely be slow (without training), thus preventing fluent responding. When programming common stimuli, generalization will be more difficult if little to no common stimuli is shared across settings.

**Rationale**

The research base in the generalization of math skills is in its infancy. Even with a plethora of general recommendations (Bender, 1996; Haring & Eaton, 1978; Stokes & Baer, 1977), the literature has only presented generalization under two conditions. First, generalization has only been observed if declarative knowledge of relevant base skills is fluent (i.e. train and hope). Second, generalization has only occurred in situations in which the topography across behaviors are similar enough to either (1) require little procedural change or (2) incorporate quick-and-simple procedures that can be used fluently with little practice. Even then generalization is unpredictable, however. If we hope to promote generalization in more meaningful situations, certain proposals that hold promise must be validated.

First, Haring and Eaton's (1978) concept of "practice" should be further analyzed in its contribution for generalization. Because programming common stimuli fundamentally reflects the IH, it is a useful strategy when applied to academic skill development. However, programming common stimuli may differentially effect generalization depending when and with what stimuli it is practiced. It has been seen that programming stimuli was unable to generalize when initiated *after* declarative knowledge is fluent (Poncy et al., 2010). However, it may be more effective if salient and functionally related stimuli are presented *simultaneously* with declarative fluency-building. By programming common stimuli within the fluency stage, additional

components are incorporated into the training environment seen in the generalized behavior. For instance, fact family triangles prompting part-part-while relationships (Poncy et al., 2010) are stimulus models that are shared across related skills like addition-to-subtraction or multiplication-to-division. The fact triangle also provides a more salient stimulus because of its "already established function for other important behaviors of the subject" (Stokes & Baer, 1977, p. 360).

Second, when programming common stimuli into the fluency-building stage, the relationship between response topographies should be taken into account. For instance, previous research indicates that generalization across related operations (addition-to-subtraction or multiplication-to-division) may be too topographically distant and therefore difficult. Programming of common stimuli is also directly involved in approaching this barrier by hoping to effect a function-based generalization gradient rather than a topography-based gradient. Because fact family triangles are stimuli that are functionally very common across inverse operations, applying them within fluency training provides a link between the trained and generalized behaviors to accommodate the differences in topography. However, the interaction between this link across various generalization gradients is unknown. To better determine this hypothesis, analysis on a skill-by-skill basis is required.

Third, the extent to which one builds such procedural fluency above declarative fluency is unknown (Poncy et al., 2010). Generalization may require a combination of declarative and procedural fluency. Furthermore, the importance of building procedural fluency may also depend on the conditions under which declarative knowledge becomes fluent (present or absent of salient/functional stimuli). Discovering answers to these

questions may inform structural changes to the IH that encourages the accuracy and fluency building of procedures as well as declarative knowledge.

The current investigation hopes to contribute to the generalization research by addressing these questions. The first goal of this study is to examine the effect of programming common stimuli within the fluency stage by incorporating salient stimuli that are functionally related to generalized outcomes. This may promote generalization by making the training environment topographically similar to the generalized outcome. This goal will be completed by comparing the differential effects of ET and CCC utilizing fact triangles on math fact fluency and the possibility of spontaneous generalization to related operations.

This investigation also hopes to identify possible interactions between these fluency-building interventions and application strategies to instruct and promote fluent procedures. Building both declarative fluency and procedural fluency in a sequential order may promote generalization, making common stimuli unneeded. Alternatively, generalization may only occur when learning procedures are simplified from early exposure to part-part-whole relationships in declarative skills. If high fluency levels are necessary but not sufficient in promoting generalization, simultaneously programming common stimuli may give additive effects to declarative fluency. Alternatively, incorporating common stimuli into fluency training may impede response rate to the extent that its incorporation be detrimental to otherwise effective fluency building. This goal will be completed by examining the interactions between intervention condition and fluency level of single-digit addition on generalization of single-digit subtraction after applying conceptual strategies.

Lastly, the prior declarative fluency X procedural fluency interaction will be further analyzed through a large-N examination to strengthen results and validate final inferences. Because generalization appears dependent on the skill under analysis, this large-N study will observe the differential effects these interactions hold between skill and age by examining third graders' ability to generalize prior single-digit multiplication skills to single-digit division skills.

**Research Questions**

Research Question 1: *Are there differences in learning rates between Cover Copy Compare utilizing fact triangles (CCC-FT) and Explicit-Timing (ET)?* While previous research (Codding et al., 2007) would contend for an intervention by instructional-level interaction, it is hypothesized that there will be a main effect for ET in improving fluency rates over CCC. Specifically, ET sessions will result in faster learning rates in the target skill of single-digit addition.

Research Question 2: *Are there differences between CCC-FT and ET in promoting generalization to related operations?* Consistent with previous research, it is hypothesized that fluency building alone will not be sufficient in promoting generalization to skills as topographically dissimilar as related operations. Therefore, it is hypothesized that both interventions will be equally ineffective in promoting subtraction skill and the inclusion of strategic programming of procedures will be needed.

Research Question 3: *Are there differences between CCC-FT and ET when interacting with procedural instruction and prior fluency?* Because this concept has not yet been addressed in the literature, no clear hypotheses can be made. However, previous research findings can help to develop two alternating hypotheses. Prior research (Poncy et al.,

2010) indicated that fluent levels of declarative and common-stimuli procedural skills alone are insufficient to promote generalized behavior. Alternatively, if "practice" by programming common stimuli while training declarative skill is more apt to promote generalized responding than declarative "drill" alone, then CCC-FT + procedure practice will result in higher levels of generalization. That is, applying already learned stimuli (fact families) to new stimuli will simplify the learning of such skills and increase their learning rate. However, because prior declarative fluency plays a necessary role, the discrete role of common stimuli can only be examined if CCC-FT and ET are equivalent prior to practicing application strategies. Finally, the role of prior fluency in the target skill can be analyzed by examining the differential generalization effects for students possessing high versus low prior fluency regarding each treatment condition.

Research Question 4: *Will these comparisons between interventions on generalization be dependent on targeted skill/age?* Because previous research suggests that skill topography affects likelihood of generalization, it is hypothesized that the generalization depicted above may be dependent on the skill analyzed. Specifically, multiplication and division are more similar operations than addition is to subtraction. Therefore, it is hypothesized that generalization from multiplication to division is more likely. However, if similar findings across skill and age are found, then the joint results will be more robust for promoting this theory of generalization. To add, the three-way (programming common stimuli X procedural practice X prior fluency) interaction to be examined by a single-case study may lack the necessary power to validly confirm findings. Therefore, the external validity of interaction effects may be greatly strengthened by examining alternative skill and ages in within a large-N context.

# CHAPTER III

# METHODOLOGY STUDY I

## Participants and Setting

Participants included one second grade class residing in a public elementary school in north central Oklahoma. The class contained 19 students, 6 male and 13 female. The ethnic background of participants was 74% White 11% Hispanic, and 11% Native American. Informed consent for participation was obtained from the school principal, teachers, and the parents of the students residing in the classes involved. All students provided personal assent for their participation.

## Materials

The materials used in this study included individualized student folders, target skill assessment packets, generalized skill assessment packets, intervention packets, an electronic timer, and assessment/intervention integrity recording sheets. Two exclusive problem sets for addition and three problem sets for subtraction were used within the experiment. Each set contained 12 single-digit computation (either addition or subtraction) problems. To increase the likelihood that set difficulty was similar the number of times a digit appears, number of same-digit problems, and number of problems possessing two digit answers across sets was matched as closely as possible. Problems containing a 0 or 1 were not used. For each addition problem set a corresponding problem set of subtraction was created. These problem sets were

controlled to match the problems of target skill by using the same fact families within an inverse operation (i.e. 3+5=8 corresponds with 8-5=3). A third subtraction probe set was added to assess spillover effects of generalized effects from both addition to subtraction skills and across subtraction skills.

**Assessment packets.** Two types of assessment packets were used in the study, one measuring the target skills (single-digit addition) and one measuring the generalized skills (subtraction). All assessment packets for addition were composed of two paper and pencil single-digit addition probes containing a one-sheet probe consisting of problem-set A and a one-sheet probe for problem set B. Each probe presented six columns of math problems, each containing eight rows. The problems were presented in a vertical format in individual boxes on the page. The class received assessment packets every session for of their targeted skill of addition. Every sixth session, the students were also assessed on three subtraction problem sets to detect any generalized effects. Each assessment probe had six alternate forms that were counterbalanced to ensure that students did not become familiarized with the order of problems. An example of an assessment probes is located in Appendix A.

**Intervention packets. Phases 1 & 2.** Two types of intervention worksheets were developed corresponding to the alternating treatments. The first of the experimental treatments used CCC procedures to increase proficiency with the addition problems of set.. Specifically, CCC procedures were used with fact triangle models to prompt correct responding (as seen in Poncy et al., 2010). The fact triangle models were presented with the problem answer at the top and the two corresponding addends on the bottom. Using this form of CCC, students engaged in a CCC intervention adapted from the guidelines in Skinner et al. (1997). Left handed students received CCC inverted worksheets with fact

triangle models on the opposite side of the response box. An example of a right-handed CCC sheet is located in Appendix B.

The second of the two treatments utilized ET procedures to increase the proficiency of the addition problems of set B. Worksheets used with the ET intervenion were identical to the format of the assessment probes with the exception that ET packets included additional sheets in case a student finished the first worksheet. The ET worksheets were identical to the assessment probes (refer to Appendix A for an example).

**Intervention Packets. Phase 3.** Two intervention probes were used in the third (generalization) phase, the think-addition worksheet and the cloze worksheet.

**"Think-addition" Worksheet.** A "think-addition" worksheet was given to students that directly corresponded to a scripted lesson that taught students how to complete the worksheet. The worksheet prompted practice highlighting part-part-whole relationships with inverse operations being covered by the lesson. These sheets provided practice of the "think-addition" strategy (Leutzinger, 2002a), in which each teaching example on the worksheet presents a standard subtraction problem and a related addition problem using a cloze format to highlight how addition and subtraction problems are related. For example, an example may be presented as "what does 8-5 equal to find out think what plus5 equals 8". Each "think-addition" worksheet presented six examples using the problems contained in the problem set being intervened upon at that treatment time (either Set A or Set B). This worksheet was completed class wide. There were six alternate forms of the "think-addition" worksheet that were used for counterbalancing. An example of this worksheet is located in Appendix C.

**Cloze worksheet.** After the conceptual lesson, students were given timed practice in cloze worksheets. While directly reflecting the "think-addition" procedures found in

Leutzinger (2002a), these worksheets were identical to those found within the ET packets with the exception of including cloze problems rather than traditional addition. Students were required to provide missing addends to complete the number sentence. For instance, instead of completing traditional fact problems like 3+5=__, students needed to fill in missing parts of the problem ___+5=8. Cloze problems were presented in a vertical format. Similar to Phase 1, the Phase 3 cloze worksheets only targeted the completion of the addition problems found within the problem set corresponding (either Set A or Set B) to the experimental treatment in session at that given time. To limit confusion, only the first/top addends were missing for all problems. This worksheet also contained six alternate forms for counterbalancing purposes. An example of this cloze worksheet is presented in Appendix D.

**Dependent Variables**

There were two dependent variables in this study. The first dependent variable included students' number of digits correct per minute (DCPM) of the targeted skill (single-digit addition). This skill was assessed every session. The second dependent variable in this study included students' DCPM of the generalized skill (single-digit subtraction). These skills were assessed every sixth session, but not intervened upon.

**Measures of Reliability and Integrity**

To ensure the reliability of scoring throughout the study, a second experimenter rescored 25% of the math probes completed by the students. Inter-scorer agreement (IA) was calculated by dividing the actual agreements on DCPM by total number of possible agreements on each probe multiplied by 100. At the conclusion of the study, the overall IA for all probes was calculated to be 99%.

Assessment and treatment integrity was evaluated daily throughout the study. This is calculated by dividing the number of procedural steps completed by the total number procedural steps required for full implementation. This was evaluated through the observation of the experimenter by a secondary experimenter. This occurred in 43% of the study sessions. Overall treatment integrity was calculated to be 97% for the entire study. Examples of assessment and treatment checklists are located in Appendix E and Appendix F.

**Experimental Design**

This study used an adapted alternating treatments design incorporating multiple-probe and changing-criterion aspects. This design was implemented to examine a second grade class as the unit of analysis, but was accommodated with idiosyncratic analyses of students' performance via multilevel modeling (Sadish, Kyse, & Rindskoph, 2013). Single-case designs formally include individual subjects to promote idiosyncratic analysis, but structuring group samples into individual units of analysis is also considered appropriate (Poncy, McCallum, & Schmitt, 2010). Utilizing a single-case design was also appropriate in instructional decision making regarding the design's changing-criterion aspect. This design was used to compare the effects of two procedures in increasing a class's math fact fluency as well as promoting generalization to non-targeted skills. By incorporating a multiple-probe procedure into the alternating treatments design, this analysis was able to compare the effectiveness of CCC and ET targeting DCPM increases and fluency growth in single-digit addition facts, as well as comparing if/when either treatment promoted generalization to single-digit subtraction. A control set was used in measuring the generalized subtraction skill to provide an anchor in fluency growth for

causal comparison between the two treatments, as well as to detect the occurrence of spillover effects/multiple treatment interference.

The study also incorporated a changing-criterion aspect into the alternating treatments design. This was included to advance procedures into a second and third treatment phase in order to determine the significance of programming common stimuli and/or fluency with prior sub-skill practice on the effectiveness of active generalization programming. Specifically, previous alternating treatments in baseline through phase 2 will receive identical treatment in phase 3 with the exception of the problem set involved. However, incorporating the final phase into the established alternating treatments can express the mediating effects of prior training types on current-or-future treatment outcomes.

**Procedures**

**Baseline assessment.** Prior to implementing treatment conditions, each student within the class was assessed over multiple days to determine a baseline level in targeted and generalized skills. Counterbalancing of the six alternate forms occurred every session for each dependent measure. Counterbalancing also occurred between dependent measures every assessment day when both were assessed. Students' scores were averaged daily to determine baseline levels of the class as the main unit of analysis for decision making toward the changing criterions. Baseline assessment occurred for the first 4 consecutive session days. Individual data were analyzed at the conclusion of the study.

**Treatment Assignment.** Once stable and equivalent baseline levels were seen across both experimental problem sets after four session days, the two problem sets were randomly assigned to one of the two treatments as its dependent variable to reflect treatment outcomes.

**Training.** The first session of each alternate treatment consisted entirely of intervention training. In the first session (implementing CCC) students were given a short lesson in how to properly engage in the practice procedure. Using only zeros and ones, the training procedure presented the procedural steps outline in Skinner et al. (1997), in which students were instructed to (1) look at the fact triangle, (2) cover the triangle, (3) write the problem and answer of one of the two possible addition problems in the space provided, (4) uncover the model and check the accuracy of the response, and (5) sub-vocally repeat and check off the problem and answer three times. This was implemented using fact triangles, in which students are instructed that the top of the triangle will always be the answer, and the bottom two numbers add together to get the top. The more in-depth script for this training task is located in Appendix H.

The probes used in the training procedure were identical to those used in the intervention phase. However, to ensure that the training procedures did not give extra practice to any problem set, all triangle models incorporated 1s as one of the addends. For instance, a triangle on the training probe might have a 5 at the top, a 4 in the bottom-left corner, and a 1 in the bottom-right corner to represent 4+1=5 and 1+4=5. An example of the CCC training worksheet is located in Appendix I. An example of the Explicit-Timing training script is located in Appendix J.

**Phase 1. Fact Fluency Building Intervention with Addition.** After baseline and training, the CCC and ET treatments were implemented to every student within an alternating-treatments design. Each intervention was conducted once daily in a class-wide format, approximately 20 min apart in the morning. The order of the daily interventions was alternated every other session day to control for order effects.

During the CCC treatment session, students worked on their CCC packets for five consecutive min. Before beginning the intervention, the experimenter read the following directions: “Here are your worksheets. You will have five minutes to complete as many math problems as you can while working on the Cover, Copy, Compare worksheet. If you finish before I say stop, raise your hand and I will give you another sheet. Ready? Begin.” These directions are outlined in the integrity checklists, located in Appendix F. The duration of the CCC treatment was approximately five minutes per session. Treatment integrity was also recorded on the integrity checklist. At the end of five minutes, the experimenter read the following: “Stop. Put your pencils down and put your packets back in your folder for me to pick up.”

During the ET treatment session, students worked on their ET packets for five timings of one minute each. Before engaging in the intervention, the experimenter read the following: “Please take out your math folder containing your Cover Copy Compare packet. The packet in front of you has addition problems for you to complete. When I say begin, start working the problems as best as you can. Begin with the first row and work across the page. Do not skip problems! But also do not guess. If you do not know the answer after you try it, you can put an X through it and move on. You will have 1 minute to do as many problems as you can before we take a break. But when I say stop, I would like everyone to put a big line after the last problem they completed. Then instead of turning to the next page, I want you to just start working on the next problem after your line. Is everyone ready?....Begin.” During this time the experimenter will record treatment integrity on the checklist. At the end of each of the first four timings, the experimenter read the following: “Stop, put your pencils down. Put a line after the last problem you completed, and get ready to continue on the next problem, but don’t start

until I say so. Make your line big enough for me to easily see. You will have another minute to do as many problems as you can again. Remember to start on the very next problem. Is everyone ready?.....Begin." At the end of the fifth timing, the experimenter read the following: "Stop. Put your pencils down and put your packet back in your folder for me to pick up." The total duration of the ET treatment sessions, including breaks between each timing, was approximately six minutes per session.

Every session of the CCC and ET treatments resulted in a new mixture of counterbalanced forms to prevent satiation/predictability as well as ensure that all problems within the set receive equal training opportunities. Counterbalancing order was recorded every session for record keeping as to prevent identical training materials being presented a few sessions apart. Therefore, each possible combination of sheets was given before presenting the first sessions' combination.

**Phase 2. Fact Fluency Building Intervention with Addition (CCC only).** When the first treatment achieved an overall fluency level of 40 DCPM for three consecutive days, it was planned a priori that the lagging intervention was to be implemented twice a day during both treatment sessions. This was to achieve equivalent levels of fluency between both interventions prior to implementing Phase 3 to control for prior fluency in sub-skills (addition) as a confounding variable in promoting generalization from the procedure fluency building intervention. Because ET achieved the fluency criterion prior to CCC, CCC was implemented twice per day for six days for a total of 12 sessions. At this time students stopped receiving the ET treatment, but continued daily assessments in the ET problem set. When both treatments were considered to show commensurate fluency levels, Phase 3 was initiated.

**Phase 3. Procedural Fluency Building Intervention.** During this phase, students engaged in the "think-addition" intervention and cloze procedure twice daily, each session employing one of the two subtraction problem sets corresponding to the two addition problem sets used with the CCC or ET treatment conditions targeting addition fluency. Students engaged in the think-addition strategy and explicitly targeted the problem sets of A and B each day with sets being practiced in isolation. Counterbalancing the problem sets every session, as conducted in the prior phases, was continued. Alternate forms were also counterbalanced in the maximum amount of combinations for both the "think-addition" worksheets and the cloze worksheets

Intervention during this phase's sessions consisted of two main steps. First, sessions within this phase began with the "think-addition" conceptual lesson focusing on part-part-whole relationships found within the inverse operations of addition and subtraction. The "think-addition" procedure lasted approximately four min. A scripted lesson reflecting this teaching phase was read aloud to the class every session that guided the students though a corresponding "think-addition" worksheet. This conceptual worksheet provided guided practice focusing on the part-part-whole relationship of inverse operations. An example of a "think-addition" lesson script is located in Appendix K.

After the brief conceptual lesson students were given timed practice in the cloze worksheets for two, one-minute timings to practice the part-part-whole relationships learned from the "think-addition" lesson. The cloze practice time only provided practice in the addition problem set that directly corresponded to the just-completed "think-addition" worksheet. This procedure of the cloze practice was identical to the past ET treatment procedure with the exception of time. Before engaging in the practice

component, the experimenter read the following: “The packet in front of you has special problems for you to complete where you need to fill in the missing number rather than finding the addition answer. You need figure out what number is missing that combines with the present number that finishes the fact. Therefore, you will write your answer on top of the problem. The arrows in the first few problems will help you remember where to put your answer. When I say begin, start working the problems as best as you can. Begin with the first row and work across the page. Do not skip problems! But also do not guess. If you do not know the answer after you try it, you can put an X through it and move on. You will have 2 timings minute each to do as many problems as you can. Is everyone ready?....Begin.” During this time the experimenter recorded treatment integrity on the checklist. At the end of the last timing, the experimenter read the following: “Stop. Put your pencils down and put your packet back in your folder for me to pick up.” An example of the cloze practice instructions are located in Appendix L.

The combined “think-addition” worksheet and the cloze worksheets were counterbalanced every session based on problem set. Each sessions counterbalanced order was recorded to ensure for the maximum amount of probe combinations before any combinations were repeated.

**Progress Monitoring**

Identical to baseline procedures, each problem set of the target skill (single-digit addition) was measured daily across Phase 1 and Phase 2. This progress monitoring occurred at the beginning of each session before the interventions were implemented to assess growth from the previous day’s treatments. For every sixth session, generalization probes for each inverse problem set were also measured along with a third unique subtraction problem set (control). The control assessment measured subtraction skills

whose fact triangles did not correspond to the ET or CCC addition problems. This third unique subtraction problem set received no prior intervention and was used to detect generalization independent of programmed common stimuli and/or prior fluency. Counterbalancing within skill (problem sets) occurred daily. Counterbalancing between skill (target and generalized) occurred on days in which both dependent variables were assessed. That is, the order of skill measured changed every twelfth session, and problem sets were counterbalanced within each skill every other session. Upon implementing Phase 3, assessment of the addition sets was ended and assessment of the generalized skill of subtraction began to occur every session.

**Analysis of Results**

Daily visual analysis was used to examine the level and trend in the class's average DCPM in single-digit addition as well as generalization to single-digit subtraction. This was done for a variety of reasons. First, daily visual analysis was needed to uphold the changing-criterion aspect of the design. Finding change in both level and trend for phase 1 would simultaneously indicate (1) if the two (or one) treatments are superior to the control, and (2) if one treatment is superior to the other. Level and trend differences between treatment groups of single-digit subtraction over time would indicate differences in their effectiveness to promote generalization from a train-and-hope perspective. Finally, results in phase 3 would indicate which, if any, treatments' ability to promote generalization is moderated by procedural fluency. This would be indicated by increases in single-digit subtraction only after the initiation of phase 3.

These visual analyses were further validated with the use of Growth Curve Modeling, a form of Hierarchical Linear Modeling (HLM) applicable to multi-phase,

within-group longitudinal analyses such as single-case designs (Van den Noortgate and Onhena, 2007). The use of Growth Curve Modeling was used to accommodate visual analysis because it allows for violations of independence, allows for examination of individual student growth over time, and permits modeling of slope and level differences across predictors including phase and treatment condition.

Two models were analyzed in the study, one model analyzing the addition outcome and a separate model analyzing the subtraction outcome. Both models included daily assessments as observation points, treatment phase, and treatment condition (level-1s) nested within individual students (level-2). The addition model compared ET and CCC on increasing addition fluency from baseline across the first treatment phase. The subtraction model analyzed these treatments' corresponding generalization occurring after implementing the "think addition" strategy as compared to an aggregate of baseline, phase 1, and phase 2 in which subtraction received no active treatment. For both models, the intercepts were centered at the last observation point of the intervention phase. The final models were defined as:

Level-1 Model:

$$E(DIVSCORE_{ti}|\pi_i) = \lambda_{ti} \log[\lambda_{ti}] = \eta_{ti} = \pi_{0i} + \pi_{1i*}(PHASE_{ti}) + \pi_{2i}*(OBSERVATION_{ti}) + \pi_{3i*}(CONDITION_{12}) + \pi_{4i}(PHASExOBSERVATIONxCONDITION) + e_{ti}$$

Level-2 Model:

$$\pi_{0i} = \beta_{00} + r_{0i}$$

$$\pi_{1i} = \beta_{10} + r_{1i}$$

$$\pi_{2} = \beta_{20} + r_{2i}$$

$$\pi_{3i} = \beta_{30} + r_{3i}$$

$$\pi_{4i} = \beta_{40} + r_{4i}$$

where $E(DIVSCORE_{ti}|\pi_i) = \lambda_{ti}$ $\log[\lambda_{ti}] = \eta_{ti}$ is equal to individual students' fluency score *i* at each time point *t* modeled with a Poisson distribution having a constant exposure with a log-link function. This logarithmic transformation was used because math CBM is considered count data that frequently assumes a Poisson distribution rather than normality (Shadish, Kyse, & Rindskopf, 2013). The parameter $\pi_{0i}$ represents the final performance occasion for each student, $\pi_{1i}$ defines the experimental phase (baseline vs treatment), $\pi_{2i}$ defines observation point over time, $\pi_{3i}$ defines the treatment condition dummy (addition) and effect (subtraction) codes, and $\pi_{4i}$ defines the interaction term between growth across phase, observations, and treatment condition. Level-2 betas ($\beta_{00,}$ $\beta_{10,}$ $\beta_{20,}\beta_{30}$) represent the fixed effects intercepts representing the average level-1 parameters over all students while the level-2 residuals ($r_{0i,}$ $r_{1i}$ , $r_{2i,}$ $r_{3i,}$ $r_{4i}$) represent their random effects that allow each case to vary from the means at random. Because the subtraction dependent variable included an additional condition, the parameters $CONDITION_{13}$ and $CONDITION_{23}$ were substituted for $CONDITION_{12}$ in a second and third conditional model to allow analysis of all pairwise comparisons across treatment conditions.

# CHAPTER IV

## RESULTS STUDY I

The distributions of both the addition and subtraction variable were assessed for normality using a Kolmogorov-Smirnov test. The results of the Kolmogorov-Smirnov test indicated that both the addition dependent variable, K-S(690)=.116, $p$<.00,1 and the subtraction dependent variable, K-S(601)=.174, $p$<.001, were significantly skewed. Therefore, all models applied a logarithmic transformation using a Poisson distribution. Unconditional models were used to determine if either treatment trend best matched either linear or quadtratic growth. The results determined that the ET treatment was best explained with a linear trajectory, $t$(252)=10.545, $p$<.001. The CCC treatment showed significant results for a quadratic trajectory, $t$(251)=2.407, $p$=.017, but was better explained with a linear trajectory $t$(252)=8.903, $p$<.001. Therefore, the results were analyzed using a linear trend in the final model.

Descriptive data of the class unit are presented in Table 1 and display addition DCPM change across the two treatment conditions and phases. Table 2 displays the results of the corresponding addition model. Figure 1 displays the class-average addition DCPM change for each treatment condition across time and phases. The baseline data for the two treatment conditions were relatively stable, having average baseline DCPM of 18.4 (ET) and 17.7 (CCC). The two corresponding subtraction problem sets plus the additional, unique, control set also indicated equivalent performance levels at 3.8 DCPM

each. Baseline stability was further validated by the model, which showed significant phase differences across baseline to the first treatment phase $t(18)=7.430$, $p<.001$.

Visual analysis of the data within the first treatment phase indicated that the introduction of both ET and CCC resulted in steady increases of DCPM over time. Visual analysis of trends for both problem sets revealed that ET resulted in a steeper trend and overall higher fluency proficiency as compared to CCC. This finding was also established from the final model. The model found a significant intercept difference between treatments $t(18)=5.001$, $p<.00125$. That is, ET's fluency level of 50.3 DCPM (increase of 32 DCPM) at the end of the first phase was higher than the final performance of 36.8 DCPM (increase of 19 DCPM) resulting from the CCC treatment. To add, the model also indicated a significant difference between treatments' slope of growth over observations, $t(18)=2.439$, $p=.025$. Specifically, the fluency growth rate of 2.15 DCPM per day resulting from ET was significantly higher than CCC's growth rate of 1.28 DCPM per day (table 1). When taking into account the total instructional time of each intervention (ET=6 mins/session; CCC=5 mins/session), ET's growth trend of approximately .35 DCPM per minute was also shown to be more efficient than CCC's growth trend of approximately .25 DCPM per minute.

To control for the fluency rate of the two treatments problem sets before implementing the generalization phase, Phase 2 was initiated where CCC was substituted in place of ET and administered for two sessions per day for an additional six days. Visual analysis of the results within this second intervention phase showed that the CCC problem set increased in overall fluency rate equal to that of ET (CCC= 46.2 DCPM; ET=48.5DCPM) at the final session. A within-subjects analysis of variance comparing final fluency scores between ET and CCC indicated no significant difference in final

fluency scores between ET and CCC, F(1,17)=1.001, $p$=.331. Although both interventions eventually showed equal effectiveness in overall fluency rate across the first two intervention phases, CCC was given 45 minutes more instructional time (135 minutes total) than ET (90 minutes total). Therefore, ET was deemed as possessing the more effective growth rate (.35 DCPM per minute) than CCC (.21 DCPM per minute) across the first two intervention phases.

The first three phases consisting of baseline, phase 1, and phase 2 were combined into an aggregate baseline for subtraction generalization, which showed equally ineffective effects across ET and CCC. The third intervention phase was implemented to examine the generalization effects that the "think addition" and cloze practice would have on both ET and CCC's corresponding subtraction problem sets. A unique subtraction problem set whose fact triangles received no prior intervention were also assessed to act as a true generalization control independent of programmed common stimuli and/or prior fluency. Two initial unconditional models were used to determine if either treatment trend best matched either linear or quadratic growth. The Results determined that the ET treatment data, $t$(180)=7.746, $p$<.001, the CCC data, $t$(180)=9.099, $p$<.001, and data of the unique control set, $t$(180)=7.664, $p$<.001, all indicated a linear trajectory. Therefore, all three conditions were analyzed using a linear trend in the final model.

Descriptive data of the class average subtraction DCPM are presented in Table 3. Table 5 displays the results of the corresponding subtraction model. Figure 1 displays the class-average subtraction DCPM change for each condition across time and phases. Results of the subtraction model indicated no significant spontaneous generalization across the aggregate baseline. However, visual analysis of the generalization phase indicated both treatment sets generalized above the control set. This was revealed in

initial level jumps across phases and increased growth rate during the generalization phase. This finding was similarly indicated in the final subtraction model that indicated significant intercept differences between treatments. Specifically, ET's final subtraction fluency level of 13.6 DCPM (increase of 7.6 DCPM), $t(18)=3.691$, $p=.002$, and CCC's final level of 17.8 DCPM (increase of 11.1 DCPM), $t(18)=-6.491$, $p<.001$, both showed significantly higher intercept levels than 8 DCPM (increase of 1.67 DCPM) of the control set. The final intercept level of CCC was also significantly higher than that of ET, $t(18)=4.499$, $p<.001$. The model also indicated a significant phaseXconditonXobservation between CCC and the control. That is, specific to the generalization phase only, the subtraction growth rate of CCC in the generalization phase (1.85 DCPM per session) was significantly higher than the .27 DCPM increase per session found in the control set, $t(18)=2.889$, $p=.010$. The subtraction growth of ET (1.26 DCPM per session) in the generalization phase was not significantly different from control, $t(18)=1.368$, $p=.188$. However, there was also not a significant difference between the generalization growth rate between ET and CCC, $t(18)=1.139$, $p=.270$. These results indicate that the addition problem set that was practiced using CCC while programming common stimuli was likely more effective in promoting generalization to subtraction.

The generalization effects of the "think addition" and cloze practice across ET and CCC was further analyzed to account for students' prior addition fluency. That is, an additional level-2 nesting variable of prior-addition-fluency was included into the model to evaluate the effects of subtraction generalization when students either achieved a prior addition fluency rate of DCPM>40 or achieved prior fluency rates below 40>DCPM during addition training in ET and CCC. This dummy coded variable was added into the level-2 model as a fixed effect for all level-1 parameters.

The descriptive results of this examination are outlined in Table 4 and displayed in Figures 2 and 3. Figure 2 displays students' subtraction performance across the aggregate baseline and generalization phase whose final addition fluency rate was less than 40 DCPM (8 of 19 students). Visual analysis of these eight students revealed negligible growth differences between phases for both treatments control. Figure 3 displays students' subtraction performance across the aggregate baseline and generalization phase whose final addition fluency rate was at or above 40 DCPM (11 of 19 students). Visual analysis of these 11 students revealed no generalization occurred across the aggregate baseline, but showed a significant growth differences during the generalization phase. This was similarly indicated in the model.

The results of the 2-level model indicated various significant groupXphaseXconditionXtime effect and are also recorded in Table 5. First, the CCC condition promoted a significantly higher growth rate in subtraction generalization than the control set, but only if prior addition fluency was 40 DCPM or above, $t(574)=2.152$, $p=.032$. Analysis of the model's intercept also revealed that group membership significantly affected the final subtraction DCPM level between CCC and control, $t(17)=4.189$, $p<.001$. That is, students who built prior addition skills to mastery levels while simultaneously programming common stimuli with CCC achieved a final subtraction fluency of 25.33 DCPM with an average growth rate of 3.01 DCPM per session, while students applying CCC but whom did not build prior addition skills to mastery only achieved a final subtraction fluency of 10 DCPM with an average subtraction growth rate of .69 DCPM per session. No such distinction was seen between the high and low group on the subtraction control set. Students achieving mastery levels addition fluency on the trained sets only increased their control subtraction fluency .5

DCPM per day on the control set to achieve a final fluency level of 10.11 DCPM, while students not achieving mastery increased control subtraction .23 DCPM per day to achieve a final fluency level of 6.13 DCPM.

A similar groupXphaseXconditionXtime interaction was found when comparing groups of students on ET to controls. The model indicated that ET promoted a significantly higher growth rate in subtraction generalization than the control set if prior fluency in addition was built to mastery, $t(574)=2.060$, $p=.040$. The intercept similarly revealed a significant group-dependent difference in final subtraction fluency performance between ET and the control depending on group, $t(17)=4.364$, $p<.001$. Specifically, students who built fluency to mastery levels with ET achieved a final subtraction fluency of 19.3 DCPM with an average growth rate of 2.1 per session, while students who used ET without gaining addition mastery only achieved a final subtraction fluency of 7.3 DCPM with an average growth rate of .35 DCPM per session. Again, these differences were not seen regarding the control set or the low fluency group.

A groupXphaseXconditionXtime interaction was found between ET and CCC for final fluency intercept. That is, high-fluency students' final subtraction fluency was significantly higher on the CCC set than the ET set, while the final fluency level of low-fluency students did not differ between the two sets, $t(17)=4.379$, $p<.001$, However, there was not a significant interaction regarding fluency growth rate slopes. Overall, these results indicate that training addition fluency with CCC while programming common stimuli results in more generalization than training with ET, but only if fluency level is trained to mastery levels.

Lastly, a descriptive analysis was conducted to examine the percent of generalization based on prerequisite skill level. This descriptive data is found in Table 4.

That is, percent of generalization was conceptualized and calculated by dividing the final division DCPM level by the prerequisite multiplication DCPM level x 100. This was conducted for the total class average and further defined across the high (addition>40 DCPM) and low (addition<40 DCPM) across the ET and CCC sets. These descriptive results revealed the following: The % generalization of the high group's ET set (19.33/64.36 x 100) was 30%, the high group's CCC set (25.33/61.73 x 100) was 41%, the low group's ET set (7.25/22.75 x 100) was 30%, and the low group's CCC set (10/21) was 48%. The descriptive results indicate that the CCC intervention, on average, increased the percent generalization by 11-18% over generalization found from ET.

# CHAPTER V

# METHODOLOGY STUDY 2

## Participants and Setting

Participants in this second study included 58 third grade students across three classrooms residing in a public elementary school in north central Oklahoma. The sample included 29 males and 29 females, being 69% Caucasian, 15% African American, 12% Hispanic, 9% African American, 2% Asian, and 2% Native American. Informed consent for participation will be obtained from the school principal, teachers, and the parents of the students residing in the classes involved.

The students were recruited for the study through their participation in a school-wide math fluency intervention passed by the school district's research institutional review board. Students in grades first through fifth participated in this system-wide intervention project titled "2-a-days," which occurred daily. As in accordance to the procedures of the project, this study disseminated "2-a-days" folders to each student containing math work sheets utilizing basic division skills. This was completed in the morning during the class-wide intervention time after school announcements.

## Materials

The materials used in this study included individualized student "2-a-days" folders, target skill assessment packets, generalized skill assessment packets, intervention packets, an electronic timer, assessment and procedural integrity recording sheets. Two exclusive problem sets for multiplication and division were used within the experiment.

Each set contained 18 single-digit multiplication or division problems. To increase the likelihood that set difficulty was similar, the number of times a digit appears, number of same-digit problems, and number of problems possessing two digit answers across sets was matched. Problems containing a zero or one were omitted. For each multiplication problem set, a corresponding generalized skill (basic division) problem set was created. The division problem sets were controlled to match the problems of multiplication probes by using the same fact families within an inverse operation (i.e. 3x5=15 corresponds with 18÷5=3).

**Assessment packets.** Two types of assessment packets were used in the study, one of which measuring the original target skill (basic multiplication) and one measuring the generalized skill (basic division). All assessment packets were composed of two paper-and pencil probes, one sheet consisting of problem-set A and one sheet for problem set B. The class received division assessment packets every session. Multiplication packets were only given prior to the first intervention session to determine fluency level to ultimately determine group.

Each assessment probe had six alternate forms for counterbalancing to ensure that students do not become familiarized with the order of problems. Each probe presented six columns of math problems, each containing eight rows. The problems were presented in a vertical format in individual boxes on the page.

**"Think-multiplication" worksheet.** The two intervention worksheets used in this phase were a "think-multiplication" worksheet and a cloze worksheet. The "think-multiplication" worksheet was given to students that directly corresponded to a scripted class-wide lesson that taught students how to complete the worksheet. This conceptual worksheet prompted practice highlighting part-part-whole relationships with inverse

operations being covered by the lesson. These sheets provided practice of the "think-multiplication" strategy (Leutzinger, 2002b), in which each teaching example on the worksheet presents a standard division problem and a related multiplication problem sing a cloze format to highlight how multiplication and division are related. For example, an example may be presented as "what does 15÷5 equal?..... to find out, think what times 5 equals 15". Each "think-multiplication" worksheet presented six examples using the problems contained in the problem set being intervened upon during the previous intervention phase. This worksheet was completed class-wide. There were six alternate forms of this worksheet for counterbalancing.

**Cloze worksheet.** After the conceptual lesson, students were given timed practice in cloze worksheets. While directly reflecting the "think-multiplication" procedures found in Leutzinger (2002b), these worksheets were identical to those found within the assessment packets with the exception of including cloze problems rather than traditional multiplication. The cloze worksheets targeted the completion of single-digit multiplication problems with problem sets corresponding to the intervention problem set. Students were required to provide missing multipliers to complete the number sentence. For instance, instead of completing traditional fact problems like 3x5=__, students were required fill in missing parts of the problem like ___x5=15. These were presented in a vertical format. Only the first multipliers for all problems were missing to limit confusion. Six alternate forms were created for counterbalancing purposes. An example of a cloze worksheet is presented in Appendix D.

**Dependent Variables**

There were two dependent variables in this study, corresponding to each type of assessment (see materials above). The first dependent variable was the mean number of

digits correct per minute (DCPM) of the prior fluency skill (basic multiplication). This skill was only used to determine fluency and group assignment. The second dependent variable in this study was the mean DCPM of the generalized skill (single-digit division). This skill was assessed prior to the implementation of every session to determine students' learning rates resulting from the intervention procedures.

**Measures of Reliability and Integrity**

To ensure the reliability of scoring throughout the study, a second experimenter re-scored 25% of all the math probes completed. Inter-scorer agreement (IA) was calculated by dividing the actual agreements on DCPM by total number of possible agreements on each probe and multiplied by 100. At the conclusion of the studies, the average of IA for all probes was calculated to be 99% overall inter-scorer agreement.

Assessment and treatment integrity was evaluated daily throughout the study. This was calculated by dividing the number of procedural steps completed by the total number procedural steps expected. This was assessed through the observation of the primary experimenter by a secondary experimenter. This occurred in 66% of the study sessions. Implementation integrity was 100% during these sessions. An example of an assessment and treatment checklist is located in Appendix E.

**Experimental Design**

This study utilized a repeated measures multivariate linear mixed model design (Molenberghs, 2007), to examine growth trajectories both within and between groups. This experimental design allows analysis across groups (prior fluency), within subjects (time), and with multiple outcome measures.

**Procedures**

**Pre-test assessment and group assignment.** Prior to this study, students received nine weeks of intervention using Cover, Copy, Compare for five minutes daily on multiplication problems only found within problem set A. The students received no practice on the second multiplication problem set, serving as a within-student control. Prior to implementing the treatment condition of this study, each student was assessed in both multiplication problem set to determine their prior fluency in the skill. The assessment consisted of three probes for problem set A and one probe for problem set B. Students' median score of their three problem set A assessments was used to categorize all students into one of three experimental groups. Students with a multiplication fluency score <19 DCPM represented the low group, students scoring between 20 and 34 DCPM represented the middle group, and students >35 DCPM represented the high group.

**Intervention phase.** During this phase, all groups engaged in the "think-multiplication" and cloze interventions once daily using problem set A. The intervention sessions consisted of two main steps. First, all students participated in the "think-multiplication" conceptual lesson focusing on part-part-whole relationships found within the inverse operations of multiplication and division. This conceptual lesson took approximately two minutes. A scripted lesson reflecting this teaching phase was read aloud to the class every session guiding the students though the "think-multiplication" worksheet.

After the brief conceptual lesson students were given timed practice in the cloze worksheets for three minutes. Before engaging in the practice component, the experimenter read the following: "Now that we completed this lesson, please take out your 2-a-days folder containing your practice packet. You will have five minutes to complete as many math problems as you can. Begin with the first problem and work

across the page, then go to the next row. If you finish before I say stop, raise your hand and I will give you another packet. Remember to try your best. Is everyone ready? Begin." During this time the experimenters recorded treatment integrity on the checklist. At the end of three minutes, the experimenter read the following: "Stop. Put your pencils down and put your packet back in your folder for me to pick up." Both the "think-multiplication" worksheet and the cloze worksheets were counterbalanced every session.

**Progress Monitoring**

Both problem set A and B of the generalization skill (basic division) was measured daily prior to each intervention session. Daily assessment of problem set A represented the growth trajectory resulting from the intervention, while daily assessment of problem set B represented the within-student control. Counterbalancing across equivalent forms as well as assessment order of problem sets occurred every session.

**Analysis of Results**

Growth Curve Modeling, a form of the general linear model using Hierarchical Linear Modeling (HLM), with restricted maximum likelihood estimators was the primary statistical technique used to evaluate trajectory differences (Raudenbush & Bryk, 2002). Growth curve modeling used with this data was capable of evaluating growth trajectories across both between (fluency group) and within (time) student conditions (Shadish, Kyse, & Rindskopf, 2013). Two models were conducted analyzing each group pairwise comparisons by using daily assessment scores (level-1) as nested within individual students (level-2). This statistical analysis controlled for violations of independence, and also permitted modeling both slope and level differences with the additional predictor of group assignment. There were 7 observation points included in each model, taken from all pre-treatment assessments plus a final posttest assessment. Observations were

centered at the final observation point to evaluate the model's intercept as the post-treatment level differences between groups. The final Growth Curve Models were defined as

Level-1 Model:

$$E(DIVSCORE_{ti}|\pi_i) = \lambda_{ti} \log[\lambda_{ti}] = \eta_{ti} = \pi_{0i} + \pi_{1i}*(OBSERVATION) + \pi_{2i}*(CONDITION) + \pi_{3i}*(OBSERVATIONxCONDITION) + e_{ti}$$

Level-2 Model:

$$\pi_{0i} = \beta_{10} + \beta_{12}*(G12_i) + \beta_{13}*(G13_i)$$

$$\pi_{1i} = \beta_{10} + \beta_{12}*(G12_i) + \beta_{13}*(G13_i)$$

$$\pi_{2i} = \beta_{10} + \beta_{12}*(G12_i) + \beta_{13}*(G13_i)$$

$$\pi_{3i} = \beta_{10} + \beta_{12}*(G12_i) + \beta_{13}*(G13_i)$$

where $E(DIVSCORE_{ti}|\pi_i) = \lambda_{ti} \log[\lambda_{ti}] = \eta_{ti}$ is equal to individual students' fluency score *i* at each time point *t* modeled with a Poisson distribution having a constant exposure with a log-link function. This logarithmic transformation was used because math CBM is considered count data that frequently assumes a Poisson distribution rather than normality (Shadish, Kyse, & Rindskopf, 2013). The parameter $\pi_{0i}$ represents the final model's intercept reflecting average ending performance levels, $\pi_{1i}$ defines the slope of growth over time, $\pi_{2i}$ defines the condition invariant effect codes between trained and untrained problem sets, and $\pi_{3i}$ defines the interaction term between condition and growth over time. The parameters $\beta_{12}$ and $\beta_{13}$ represent group membership effect codes that allow comparisons of student trajectories between experimental groups based on prior-fluency of multiplication. The parameter $\beta_{23}$ was substituted for $\beta_{13}$ in a second conditional model allowing for analysis of all pairwise comparisons. Group differences were modeled at Level-2 for $\pi_0$, $\pi_1$, $\pi_2$, and $\pi_3$ allowing for comparisons of slopes and final

performance and between groups and within assessment condition. The high-fluency group is referred to as Group 1 (G1), the instructional-fluency group as Group 2 (G2), and the low-fluency group as Group 3 (G3).

# CHAPTER VI

## RESULTS STUDY 2

The distribution of the division measure was assessed for normality using a Kolmogorov-Smirnov test. The results of the Kolmogorov-Smirnov test indicated the dependent variable was significantly skewed (K-S(782)=.131, $p<.001$). Therefore, the two models applied a logarithmic transformation using a Poisson distribution because math CBM. For 58 participants across fourteen observation assessments (812 total data points), there were 44 missing datum points distributed evenly across groups. Two initial unconditional models were first tested to examine whether the results were best explained by either a linear or quadratic growth trajectory. The results indicated that the model was best explained with a linear trajectory, $t(57)=5.406$, $p<.001$.

Descriptive data from the three groups and problem sets are presented in Table 6 and displays the group average DCPM for each problem set across time for each assessment condition. Table 7 displays the results from the corresponding model. Figure 4 displays the average DCPM trajectories for each group and their within-student assessment condition across time. Raw data of students' DCPM are more interpretively useful than log-link coefficients, and were therefore used to supplement the model's interpretation. A one-way multivariate analysis of variance on the initial assessment from the first session indicated that that the three groups were not equal at the initial assessment, $F(2,54)=.14.076$, $p<.001$. This was expected, but this confound was

somewhat accommodated experimentally by incorporating the problem sets variable nested within students to serve as an intra-individual control (refer to methods above).

The growth curve model revealed several significant groupXconditionXtime interactions. That is, there was significant intercept and slope differences over time between the trained and untrained problem sets, but this affected the groups differently. First, The model indicated that the high group had significantly higher growth rate slope differences between their trained and untrained problem sets compared to condition differences in the low group, $t(603)=2.782$, $p=.006$. That is, the high-fluency group's average growth rate of 2.94 DCPM per session on the fluent problem set was significantly higher than their untrained problem set (1.05 DCPM per session), while growth differences of the low group's trained (.75 DCPM per session) and untrained (.24 DCPM) problem set showed an undifferentiated pattern.. A similar interaction was seen regarding final fluency intercepts, as the difference in the two problem sets of the high group was significantly larger than that found within the low group, $t(55)=10.624$, $p<.001$. The high group's final fluency level on the trained problem set (30.6 DCPM) was significantly higher than their final fluency level on untrained problems (16.5 DCPM), but no distinction was seen between the low group's final fluency on trained (8.8 DCPM) and untrained problems.

The high group also showed similar differences compared to the middle (instructional) group. Specifically, while the high group's growth rate deviation between the two problem set was significant, the middle group's growth rate on the trained (.91 DCPM per session) and untrained (.26 DCPM per session) problem set conditions did not differ, $t(603)=2.140$, $p=.033$. Similarly, the high group's final fluency intercept were larger across the problem set conditions than the difference between the middle group's

trained (14.7 DCPM) and untrained (7.9 DCPM) sets, which also reflects the three-way interaction, $t(55)=6.435$, $p<.001$.

The model found no significant groupXconditionXtime interaction between the slopes of the middle and low groups, as both groups showed little growth differences across the trained and untrained problem conditions. However, the model did indicate a significant groupXconditionXtime effect regarding final fluency intercepts, $t(55)=4.786$, $p<.001$. Specifically, the level difference between the middle group's trained and untrained problem set was larger than the differences seen within the low group.

Because there was a significant groupXconditionXtime interaction, two follow-up split-plot analyses were conducted by examining the problem set conditions separately in two separate conditional model. This was conducted to examine the simple main effects across groups for trained and untrained problem sets in pairwise fashion. This was completed to help accommodate the quasi-experimental nature of group assignment by controlling for possible generalization not occurring from prior fluency (i.e. individual ability). This growth curve model was defined as:

Level-1 Model:

$$E(DIVSCORE_{ti}|\pi_i) = \lambda_{ti} \log[\lambda_{ti}] = \eta_{ti} = \pi_{0i} + \pi_{1i}*(OBSERVATION)$$

Level-2 Model:

$$\pi_{0i} = \beta_{10} + \beta_{12}*(G12_i) + \beta_{13}*(G13_i)$$

$$\pi_{1i} = \beta_{10} + \beta_{12}*(G12_i) + \beta_{13}*(G13_i)$$

$$\pi_{2i} = \beta_{10} + \beta_{12}*(G12_i) + \beta_{13}*(G13_i)$$

$$\pi_{3i} = \beta_{10} + \beta_{12}*(G12_i) + \beta_{13}*(G13_i)$$

Where the parameter $\pi_{0i}$ represents the final performance occasion for each student and $\pi_{1i}$ defines the slope of growth over time. Group differences were modeled at Level-2 for

$\pi_0$ and $\pi_1$, allowing for comparisons simple main effects of slopes and final performance between groups.

The results of the split plot growth curve models are presented in table 8. The first split plot model indicated that the trained problem set showed various simple main effects. When comparing group differences within the trained problem set, the high group showed significantly higher intercepts than those of both the low, $t(55)=9.017$, $p<.001$, and middle group $t(55)=5.574$, $p<.001$. The high group also showed significantly higher growth rate slopes over time compared to the low, $t(330)=3.711$, $p<.001$, and middle groups, $t(330)=2.590$, $p<.001$. The model also indicated that the middle group possessed a significantly higher final intercept compared to the low group, $t(55)=3.923$, $p<.001$. There were no differences in slope between the middle and low group on the trained problem set.

When comparing group differences within the untrained problem set, the high group showed a significantly higher intercept than the low, $t(55)=7.904$, $p<.001$, and middle group, $t(55)=4.236$, $p<.001$. The middle group also possessed a higher intercept than the low group, $t(55)= 4.147$, $p<.001$. However, the high group's untrained problem set did not show any significant slope differences over either group. That is, although students in the high group started with higher basic division fluency compared to students in the middle and low groups, they did not increase their basic division DCPM in the untrained problem set for the entirety of the study. In fact, the growth rate of the high students' division using untrained problems more closely reflected the division growth rate of middle and low students' trained problem set. That is, the division increase of 1.05 DCPM per session of the high group in the untrained set was closer to the division fluency increase over time of the middle (.91 DCPM per session) and low (.75 DCPM

per session) groups trained set rather than the growth seen on their own trained set (2.94 DCPM per session). This provides some support that the prior fluency level of multiplication skills held a larger impact than that of intra-individual ability.

Lastly, a descriptive analysis was conducted to examine the percent of generalization obtained by groups based on prerequisite skill level in multiplication fluency. That is, percent of generalization was conceptualized and calculated by dividing the final division DCPM level by the prerequisite multiplication DCPM level x 100. These descriptive results revealed the following: The % generalization of the high group's fluent set (30.64/54.13 x 100) was 57%, the high group's control set (16.5/29.86 x 100) was 55%, the instructional group's fluent set (14.71/26.75 x 100) was 55%, the instructional group's control set (7.81/13.32) was 59%, the low group's fluent set (8.81/13.14) was 67%, and lastly the low group's control set (3.57/7.86) was 45%. The percent generalization was determined to be comparable across all groups and sets.

# CHAPTER VII

# DISCUSSION

The social and political context of school mathematics performance has raised awareness towards increasing evidence-based practice for intervention and remediation. Because learning problems are dualistic (Skinner, 1996), failing to evaluate interventions based on instructional time can mask true effects. Focusing on learning rate requires direct interventions comparison rather than comparing against control conditions only (Poncy, Skinner, & Jaspers, 2007; Skinner, Belfiore, & Watson, 2002). Intervention effectiveness may also be improved if it is able to improve multiple skills (i.e. generalization), especially if doing so does not require more instructional time. Generalization should therefore be a highly valued intervention outcome metric. While occasionally observed from a train and hope perspective, generalization has yet to be empirically observed from active training of declarative and procedural intervention (Poncy et al., 2010).

The current work hoped to examine the role of programming common stimuli and prior fluency of prerequisite subskills to actively promote generalization. Specifically, the first study compared the two interventions of Explicit Timing (ET) and Cover Copy Compare (CCC) on basic math facts to (1) directly compare the learning rate of each in increasing fluency, (2) compare their effects on train and hope toward generalization to basic subtraction skills, and (3) examine their interaction effects between programming

common stimuli, training procedural skills, and prior fluency rate in promoting generalization of addition skills to subtraction skills. The second study extended the external validity of these questions though their supplemental analysis in a group context.

**Summary of Findings**

This examination in total showed some very interesting findings regarding the research questions above. First, the results of the first study determined that Explicit Timing resulted in higher learning rates in building basic addition fluency over that of Cover Copy Compare using fact triangles. The experimental design in which this was determined also expresses the importance of examining intervention efficiency over level change only. Even though the ET intervention took 20% more time per session, it was superior in learning rate (DCPM/per minute) as well as overall fluency level in the first intervention phase. This infers that ET is, overall more effective than CCC. However the inclusion of the second intervention phase while neglecting instructional time would wrongfully indicate that both interventions are of equal effectiveness in increasing fluency performance. That is, because both interventions eventually achieved equivalent fluency scores, they reflect similar effectiveness if neglecting the duration of each. Because we accounted for instructional time on the horizontal axis by incorporating phase 2, visual analysis of the graphical results clearly showed that the CCC intervention was given much more time than ET. Presenting intervention duration in this fashion is rarely seen and traditional alternating treatment designs do not always expose instructional time in such a way. Therefore, this study's unique method of examining instructional time by incorporating a changing-criterion element into its alternating treatment design is considered a methodological strength.

Second, it was determined that both interventions were equally ineffective in promoting generalization to basic subtraction skills taking a train and hope perspective. This finding is consistent with previous research (Poncy et al., 2010). However, generalization was seen once procedural instruction and practice was implemented. This is inconsistent with prior research (Poncy et al., 2010) which showed that even pairing fluent declarative and procedural skills did not promote generalization. Upon further analyses between students (for both conditions) who achieved fluent levels of addition and those who did not revealed that generalization of subtraction only occurred if addition achieved mastery levels. Therefore, this study confirms prior understandings into the Instructional Hierarchy which contends that fluency is necessary but not sufficient for promoting generalization (Haring & Eaton, 1978). The external validity of this contention was supported in the second study which indicated that generalization is highly contingent on fluency levels of prerequisite subskills, perhaps above that of individual ability.

Third, although both interventions in study 1 showed generalization after procedural instruction and practice, the CCC intervention did show superior generalization effects, especially if prerequisite skills achieved mastery levels. This was assumed to occur because CCC using fact triangles programmed common stimuli (Stokes & Baer, 1977). Specifically, CCC incorporating fact triangles exposed students to unique stimuli (the fact triangles themselves) that were functionally useful across the basic addition skills and the generalized skill of basic subtraction. One hypothesis into how this occurred is that incorporating fact triangles enhanced generalization by linking the topographically dissimilar trained and generalized skills with a function similar across these skills (Codding, Archer, & Connell, 2010; Codding, Chan-Iannetta, Palmer, &

Lukito, 2009b). Yet because no generalization was seen prior to procedural instruction/practice (even if fluent in the trained skill), it is likely that this is not the sole reason for generalization. Rather, the results indicated that programming common stimuli into fluency training (CCC) provided additive effects to the "Think Addition" strategy beyond regular fluency building (ET). Therefore, while this reiterates that fluency of declarative knowledge is necessary but not sufficient for generalization, building fluency in one's knowledge of part-part-whole relationships simultaneously with declarative knowledge may be required to most effectively apply such procedures for generalization.

Prior research has supported the notion that generalization requires fluency of all component subskills comprising a taxonomy required to perform an expected generalized behavior (Binder, 1996). In this way, knowledge of part-part-whole relationships may be somewhat conceptualized as an important subskill in need of mastery just as declarative skills (such as addition facts) are. Without all necessary subskills, perhaps addition skills not exposed to these functionally-useful stimuli do not put students at a truly "instructional range" for generalizing behavior and skills. Because student's fluency base in part-part-whole relationships was not measured during CCC training, this assumes some presence of implicit learning in the present study. Yet overall, the interaction between procedural instruction and prior fluency building techniques indicate that programming common stimuli was most effective in promoting generalization due to a combination between all three variables. That is, building procedural skills to fluency in addition to declarative skills results in some generalization, but procedural fluency will be built more effectively if fluent declarative skills are highly utilized within its application. This is one example distinguishing between drill and practice (Haring & Eaton, 1978). Therefore, if fluency of declarative skills is necessary *but not* sufficient for

generalization, our results indicate that programming common stimuli is a potential variable to make fluency necessary *and* sufficient.

**Limitations**

Although this study presents several practical and theoretical implications, there are several limitations found herein to which our inferences require caution. The first applies to the external validity of programming common stimuli as an effective strategy to actively promote academic generalization. Although this study is one of the first to experimentally observe systematic techniques for generalization of basic math skills, the conclusions are initial at best. Study 1 included only 19 students, which was further divided into groups of eight and 11 students when comparing between students of high and low fluency. To add, our conclusions are only directed to generalization found from basic addition to basic subtraction skills on second grade students. Future research would thus benefit by examining these initially successful procedures across student age and a variety of targeted skills. Study 2 provided external validity into the importance of mastery in prerequisite skills, but did not examine the role of programming common stimuli on a larger scale. Because previous findings from Poncy et al. (2010) failed to detect generalization from procedures nearly identical to those applied here, it is inferred that successful generalization found in Study 2 may have resulted from utilizing older students who may have been provided more exposure to fact triangles throughout their educational experiences. This cannot be validated, however. Nevertheless, the conclusions from both studies provide initial validity for the use of programming stimuli and strong findings supporting the importance of fluency in prerequisite declarative skills.

Second, attempting to simultaneously examine programming common stimuli and instructional time while controlling for prior fluency rates of prerequisite declarative skills potentially creates experimental confounds. As stated above, the second intervention phase in Study 1 (CCCx2) was included into this study to control for fluency levels prior to administering the generalization procedures. Including this phase also was successful in expressing the importance of examining learning rate above level change as an intervention outcome. However, controlling for fluency in this way prevented the control of instructional time. Because CCC was given 45 minutes more time, this extra time cannot be partialed out of the generalized effects. That is, it is possible that CCC was only superior to ET in generalization because of the extra 45 minutes students were exposed to the intervention and/or unique problem sets and not necessarily from programming common stimuli. Because a similar weakness would be present if attempting to examine generalization effects across treatments holding unequal fluency (and eliminating phase 2 of the CCC intervention), this design was chosen as the best of two methods for an initial examination of our hypothesis. Future research should replicate the procedures of this study while controlling for time instead of controlling for ending fluency level.

Finally, the "Think Addition" and cloze practice procedures found during the generalization phase of the first study were combined, disallowing us to partial out each technique's effect towards overall generalization. These were combined because prior research found null effects for either in isolation or in combination (Poncy et al., 2010). However, the effectiveness of their combination in this study warrants their separation in future studies.

**Implications and Future Directions**

This study holds several important clinical implications and provides future research directions. First, this work accommodates the present need for empirical comparisons of efficiency between interventions. Future studies should continue to push this need of determining not only what works, but what works best.

Second, this study confirms previous understandings of the IH by finding that fluency is necessary for generalization, but perhaps not sufficient. Yet being one of the first studies to show systematic building of basic math generalization through programming common stimuli during the fluency phase, the static structure of the IH may need to be modified. Specifically, our observed interaction between procedural and declarative fluency support Poncy et al (2010)'s notion that the IH may be better conceptualized as multilateral regarding skill functions and topographies. This study takes this concept even further to raise questions regarding the pinpointing and/or curricular scope of building procedural fluency. Is incorporating common stimuli like fact triangles into fluency practice best conceptualized as building a prerequisite subskill of some overarching procedural knowledge, like that found in "think addition/multiplication," or merely effective by making addition more functionally and topographically similar to subtraction?

Finally, this work hopes to be succeeded by questions informing that what works best depends on *what outcome you seek.* Just as examining learning/achievement level while neglecting efficiency can bias decisions about the intervention outcomes, examining efficiency without examining multivariate effects (i.e., response generalization) may also mask effects. Future research should continue this effort, but be mindful of the negative correlation between instructional efficiency and breath of effect. Specific to this study, future research should examine the effects of programming

common stimuli using CCC and fact triangles in promoting generalization above that of ET while controlling for instructional time.

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Table 1.

*Study 1 descriptive statistics for addition outcome across group, problem set, and time.*

| Phase | Initial | | | Phase 2Final | | | Growth Rate |
|---|---|---|---|---|---|---|---|
| | *n* | *m* | *sd* | *n* | *m* | *sd* | (DCPM/Session) |
| Baseline | | | | | | | |
| ET | 19 | 18.2 | 6.3 | 19 | 18.0 | 6.6 | -.07 |
| CCC | 19 | 17.6 | 6.8 | 19 | 17.5 | 7.6 | -.03 |
| Phase 1 | | | | | | | |
| ET | 19 | 25.2 | 10.2 | 19 | 50.3 | 21.4 | 2.15 |
| CCC | 19 | 23.5 | 11.7 | 19 | 36.8 | 21.2 | 1.28 |
| Phase 2 | | | | | | | |
| ET | 19 | 44.0 | 19.3 | 19 | 48.6 | 26.0 | .76 |
| CCC | 19 | 37.9 | 20.6 | 19 | 46.2 | 26.3 | 1.38 |

*Note.* Time columns based on initial and final datum point within each phase.

Table 2.

*Study 1 results of addition growth curve analysis*

| Fixed Effects | Coefficient | *SE* | *t* | *df* | *p* |
|---|---|---|---|---|---|
| Intercept ($\beta_{00}$) | 3.408 | .149 | 22.844 | 18 | < .001 |
| phase ($\beta_{10}$) | 0.290 | .039 | 7.430 | 18 | <.001 |
| Observations ($\beta_{20}$) | 0.015 | .004 | 4.155 | 18 | <.001 |
| Condition ($\beta_{30}$) | 0.153 | .031 | 5.001 | 18 | <.001 |
| PhaseX Condition X Observations ($\beta_{40}$) | 0.006 | .003 | 2.439 | 18 | =.0025 |
| **Random Effects** | **sd** | ***Variance Component*** | **$\chi^2$** | ***df*** | ***p*** |
| Intercept ($r_{0i}$) | 0.628 | .394 | 265.685 | 18 | < .001 |
| Phase ($r_{1i}$) | 0.117 | .014 | 31.482 | 18 | =.025 |
| Observations ($r_{2i}$) | 0.015 | .002 | 169.830 | 18 | < .001 |
| Condition ($r_{3i}$) | 0.072 | .005 | 15.828 | 18 | >.500 |
| Phase X Condition X Observations ($r_{4i}$) | 0.009 | .001 | 55.260 | 18 | <.001 |

*Note.* Coefficient are standardized based on Poisson log-link function.

Table 3.

*Study 1 descriptive statistics for subtraction outcome across treatment condition, phase, and time.*

| Phase | Initial | | | Final | | | Growth Rate |
|---|---|---|---|---|---|---|---|
| Condition | *n* | *m* | *sd* | *n* | *m* | *sd* | |
| Baseline | | | | | | | |
| Control | 19 | 3.8 | 2.4 | 19 | 6.3 | 2.9 | .09 |
| ET | 19 | 3.8 | 1.5 | 19 | 6.0 | 2.6 | .06 |
| CCC | 19 | 3.8 | 2.1 | 19 | 6.7 | 2.9 | .07 |
| Gen. Phase | | | | | | | |
| Control | 19 | 8.4 | 3.6 | 19 | 8.0 | 4.1 | .27 |
| ET | 19 | 8.2 | 4.3 | 19 | 13.6 | 9.5 | 1.26 |
| CCC | 19 | 10.2 | 5.6 | 19 | 17.8 | 12.8 | 1.85 |

*Note.* Time columns based on initial and final datum point within each phase.

Table 4

*Study 1 descriptive statistics of subtraction generalization of CCC and ET across groups and instructional-time metric*

| Group | Final Fluency | Total increase | Increase per Gen. Session | Increase per Minute | % Generalization |
|---|---|---|---|---|---|
| CCCtotal | 17.8 | 11.10 | 1.85 | 0.063 | 38% |
| CCChigh | 25.3 | 18.06 | 3.01 | 0.102 | 41% |
| CCClow | 10.0 | 4.16 | 0.69 | 0.023 | 48% |
| Ettotal | 13.6 | 7.60 | 1.26 | 0.058 | 28% |
| EThigh | 19.3 | 12.51 | 2.1 | 0.095 | 30% |
| ETlow | 7.25 | 2.13 | 0.35 | 0.016 | 30% |

*Note.* CCC=177 minutes, ET=132 minutes

Table 5.

*Study 1 combined results of Level-1 and Level-2 subtraction growth curve analyses.*

| Level-1 Effects<br>Level-2 Effects | Coefficient | *SE* | *t* | *df* | *p* |
|---|---|---|---|---|---|
| Intercept ($\beta_{12}$) | 2.078 | 0.105 | 19.806 | 18 | < .001 |
| Group ($\beta_{12}$) | 0.546 | 0.180 | 3.044 | 17 | < .001 |
| Intercept ($\beta_{13}$) | 2.033 | 0.101 | 20.086 | 18 | < .001 |
| Group ($\beta_{13}$) | 0.521 | 0.180 | 2.901 | 17 | = .010 |
| Intercept ($\beta_{23}$) | 2.063 | 0.103 | 19.988 | 18 | < .001 |
| Group ($\beta_{23}$) | 0.545 | 0.180 | 3.036 | 17 | = .007 |
| Phase ($\beta_{12}$) | 0.195 | 0.082 | 2.389 | 18 | = .028 |
| Group ($\beta_{12}$) | 0.208 | 0.053 | 3.931 | 574 | < .001 |
| Phase ($\beta_{13}$) | 0.210 | 0.082 | 2.561 | 18 | = .020 |
| Group ($\beta_{13}$) | 0.206 | 0.053 | 3.879 | 574 | < .001 |
| Phase ($\beta_{23}$) | 0.198 | 0.082 | 2.433 | 18 | = .026 |
| Group ($\beta_{13}$) | 0.212 | 0.053 | 4.008 | 574 | < .001 |
| Observation ($\beta_{12}$) | 0.008 | 0.001 | 6.224 | 18 | < .001 |
| Group ($\beta_{12}$) | .001 | 0.002 | .000 | 574 | 1.000 |
| Observation ($\beta_{13}$) | 0.007 | 0.001 | 6.288 | 18 | < .001 |
| Group ($\beta_{13}$) | 0.001 | 0.001 | 0.260 | 574 | =. 795 |
| Observation ($\beta_{23}$) | 0.007 | 0.001 | 6.322 | 18 | < .001 |
| Group ($\beta_{23}$) | 0.001 | 0.001 | 0.021 | 574 | = .983 |
| Condition ($\beta_{12}$) | 0.080 | 0.022 | 3.691 | 18 | = .002 |

| | | | | | |
|---|---|---|---|---|---|
| Group ($\beta_{12}$) | 0.072 | 0.055 | 1.305 | 574 | = .192 |
| Condition ($\beta_{13}$) | 0.234 | 0.036 | 6.491 | 18 | < .001 |
| Group ($\beta_{13}$) | 0.029 | 0.056 | 0.528 | 574 | = .598 |
| Condition ($\beta_{23}$) | 0.149 | 0.033 | 4.499 | 18 | < .001 |
| Group ($\beta_{23}$) | 0.042 | 0.056 | 0.747 | 574 | = .455 |
| Phase X Condition X Observation Slope ($\beta_{12}$) | 0.001 | 0.001 | 1.368 | 18 | = .188 |
| Group ($\beta_{12}$) | 0.013 | 0.006 | 2.053 | 574 | = .041 |
| Phase X Condition X Observation Slope ($\beta_{13}$) | 0.002 | 0.001 | 2.889 | 18 | = .010 |
| Group ($\beta_{13}$) | 0.013 | 0.006 | 2.154 | 574 | = .032 |
| Phase X Condition X Observation Slope ($\beta_{23}$) | .001 | .001 | 1.139 | 18 | = .270 |
| Group ($\beta_{23}$) | 0.001 | 0.006 | 0.134 | 574 | = .893 |

| Level-1 Random Effects | sd | *Variance Component* | $\chi^2$ | *df* | *p* |
|---|---|---|---|---|---|
| Intercept ($r_{12}$) | 0.369 | .136 | 50.007 | 18 | < .001 |
| Intercept ($r_{13}$) | 0.376 | .142 | 68.606 | 18 | < .001 |
| Intercept ($r_{23}$) | 0.372 | .138 | 56.277 | 18 | < .001 |
| Phase ($r_{12}$) | 0.249 | .062 | 32.801 | 18 | = .018 |
| Phase ($r_{13}$) | 0.284 | .080 | 47.986 | 18 | < .001 |
| Phase ($r_{23}$) | 0.263 | .069 | 38.313 | 18 | =.004 |
| Observations ($r_{12}$) | 0.003 | .001 | 21.804 | 18 | = .240 |

| | | | | | |
|---|---|---|---|---|---|
| Observations ($r_{13}$) | 0.002 | .001 | 27.100 | 18 | = .077 |
| Observations ($r_{23}$) | 0.003 | .001 | 23.675 | 18 | = .166 |
| Condition ($r_{12}$) | 0.133 | .017 | 68.606 | 18 | < .001 |
| Condition ($r_{13}$) | 0.133 | .018 | 73.197 | 18 | < .001 |
| Condition ($r_{23}$) | 0.112 | .012 | 51.950 | 18 | < .001 |
| ConditionX Phase X Observations ($r_{12}$) | 0.001 | .001 | 6.431 | 18 | >.500 |
| ConditionX Phase X Observations ($r_{12}$) | 0.001 | .001 | 9.646 | 18 | >.500 |
| ConditionX Phase X Observations ($r_{12}$) | 0.009 | .001 | 11.383 | 18 | >.500 |

*Note.* Coefficient are standardized based on Poisson log-link function. *Note.* $\beta_{12}$ & $r_{12}$= Control vs. ET, $\beta_{13}$ & $r_{13}$ = Control vs. CCC, $\beta_{23}$ & $r_{23}$= ET vs. CCC.

*Note.* Slopes reflect phaseXcondition interaction parameter.

*Note.* Coefficient based on Poisson log-link function.

Table 6

*Study 2 descriptive statistics across fluency group, assessment condition set, and time.*

| Fluency Group | Initial | | | Final | | | Growth Rate |
|---|---|---|---|---|---|---|---|
| Problem Set | *n* | *m* | *sd* | *n* | *m* | *sd* | (DCPM/Session) |
| Low (G3) | | | | | | | |
| Trained | 15 | 4.3 | 4.4 | 15 | 8.8 | 4.3 | .75 |
| Untrained | 15 | 3.3 | 2.6 | 15 | 3.6 | 3.1 | .24 |
| Instructional (G2) | | | | | | | |
| Trained | 22 | 9.3 | 5.1 | 22 | 14.7 | 5.6 | .91 |
| Untrained | 22 | 6.2 | 3.0 | 22 | 7.9 | 4.7 | .26 |
| High (G1) | | | | | | | |
| Trained | 21 | 13.0 | 5.3 | 21 | 30.6 | 6.7 | 2.94 |
| Untrained | 21 | 10.2 | 4.3 | 21 | 16.5 | 7.5 | 1.05 |

*Note.* Time columns based on initial and final datum point within each phase.

Table 7.

*Study 2 results of growth curve analysis.*

| Fixed Effects | Coefficient | *SE* | *t* | *df* | *p* |
|---|---|---|---|---|---|
| Intercept ($\beta_{12}$) | 7.382 | .096 | 76.496 | 768 | < .001 |
| Intercept ($\beta_{13}$) | 9.967 | .104 | 94.967 | 768 | < .001 |
| Intercept ($\beta_{23}$) | 2.584 | .095 | 27.015 | 768 | < .001 |
| Slope ($\beta_{12}$) | .146 | .012 | 11.743 | 768 | < .001 |
| Slope ($\beta_{13}$) | .269 | .013 | 19.812 | 768 | < .001 |
| Slope ($\beta_{23}$) | .122 | .012 | 9.920 | 768 | <. 001 |

*Note.* $\beta_{12}$ = High-fluency vs. Instructional-fluency, $\beta_{13}$ = High-fluency vs. Low-fluency, $\beta_{23}$ = Instructional-fluency vs. Low-fluency.

*Note.* Slopes reflect observationXcondition interaction parameter.

*Note.* Coefficients based on Poisson log-link function.

Table 8.

*Study 2 results of split plot growth curve analyses.*

| Trained Problem Set | Coefficient | *SE* | *t* | *df* | *p* |
|---|---|---|---|---|---|
| Intercept ($\beta_{12}$) | .848 | .152 | 5.574 | 55 | < .001 |
| Intercept ($\beta_{13}$) | 1.432 | .159 | 9.017 | 55 | < .001 |
| Intercept ($\beta_{23}$) | .585 | .149 | 3.923 | 55 | < .001 |
| Slope ($\beta_{12}$) | .025 | .010 | 2.590 | 330 | = .010 |
| Slope ($\beta_{13}$) | .030 | .008 | 3.711 | 330 | < .001 |
| Slope ($\beta_{23}$) | .005 | .010 | 0.505 | 330 | = .614 |
| Untrained Problem Set | Coefficient | *SE* | *t* | *df* | *p* |
| Intercept ($\beta_{12}$) | .715 | .169 | 4.236 | 55 | < .001 |
| Intercept ($\beta_{13}$) | 1.408 | .178 | 7.904 | 55 | < .001 |
| Intercept ($\beta_{23}$) | .694 | .167 | 4.147 | 55 | <.001 |
| Slope ($\beta_{12}$) | .007 | .014 | .543 | 330 | = .587 |
| Slope ($\beta_{13}$) | .017 | .013 | 1.286 | 330 | = .199 |
| Slope ($\beta_{23}$) | .009 | .011 | 0.881 | 330 | = .379 |

*Note.* $\beta_{12}$ = High-fluency vs. Instructional-fluency, $\beta_{13}$ = High-fluency vs. Low-fluency, $\beta_{23}$ = Instructional-fluency vs. Low-fluency.

*Note.* Slopes reflect observationXcondition interaction parameter.

*Note.* Coefficients based on Poisson log-link function.

Figure 1.

*Study 1 group fluency growth in DCPM in basic addition problem sets and related subtraction problem sets across all four study phases.*

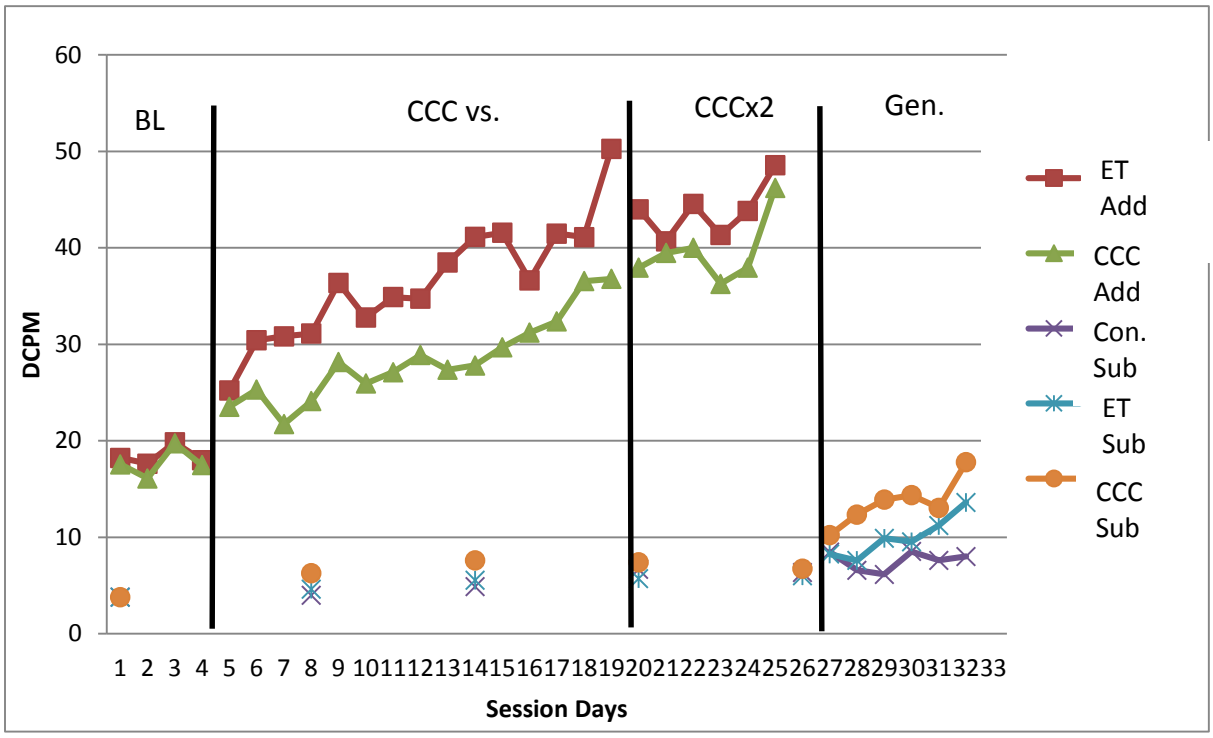


Figure 2.

*Study 1 generalization fluency growth in subtraction DCPM of students achieving <40 in basic addition DCPM before and after generalization phase.*

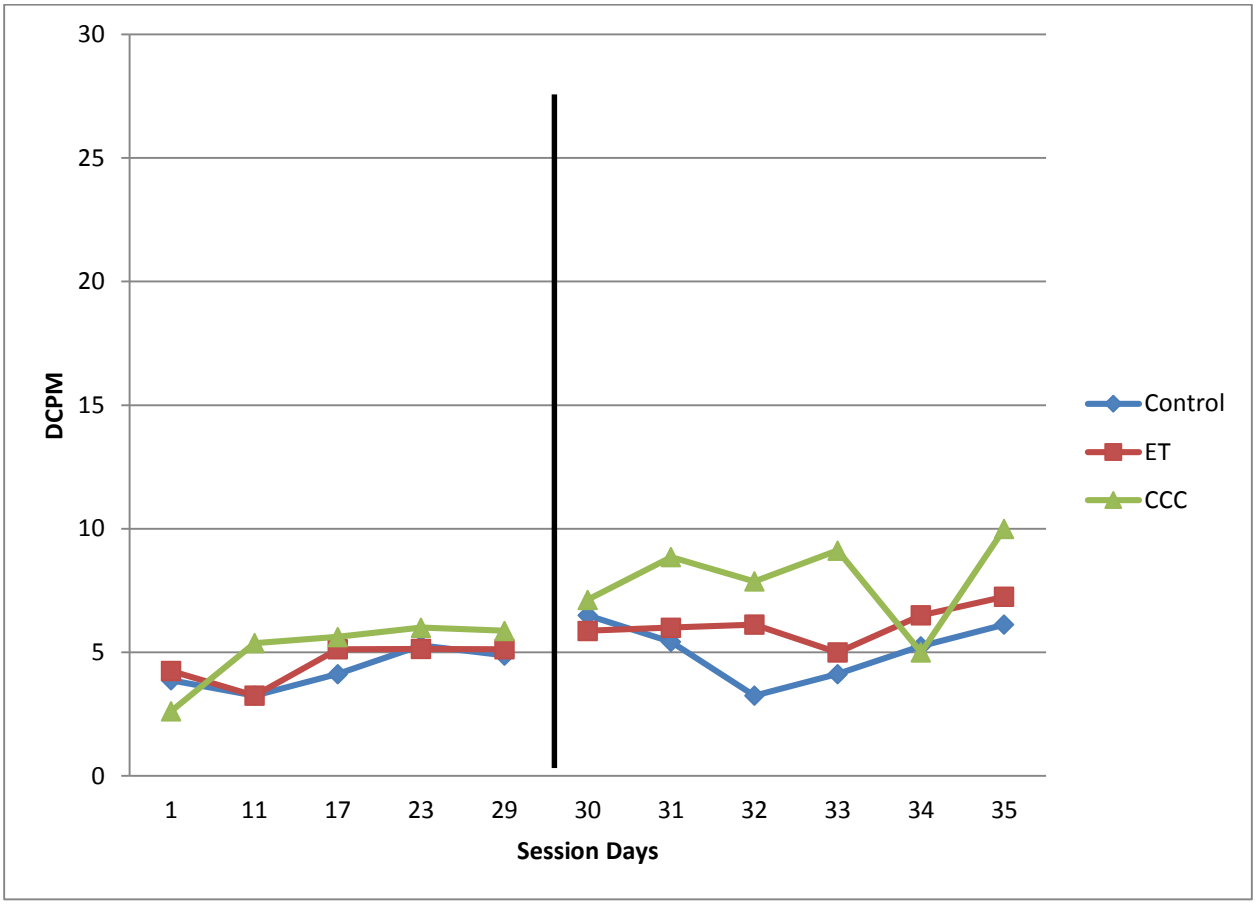


Figure 3.

*Study 1 generalization fluency growth in subtraction DCPM of students achieving >40 DCPM in basic addition skills before and after generalization phase.*

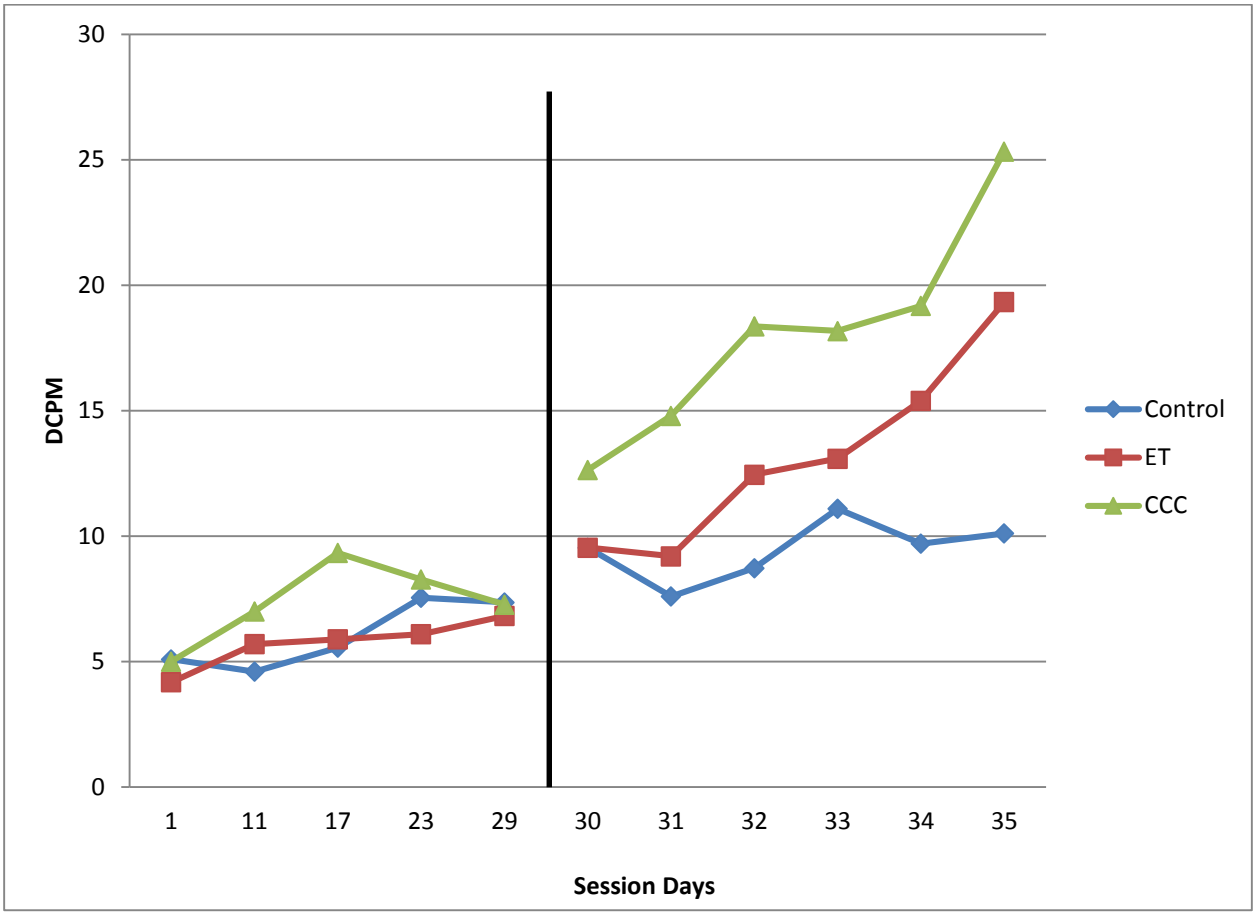
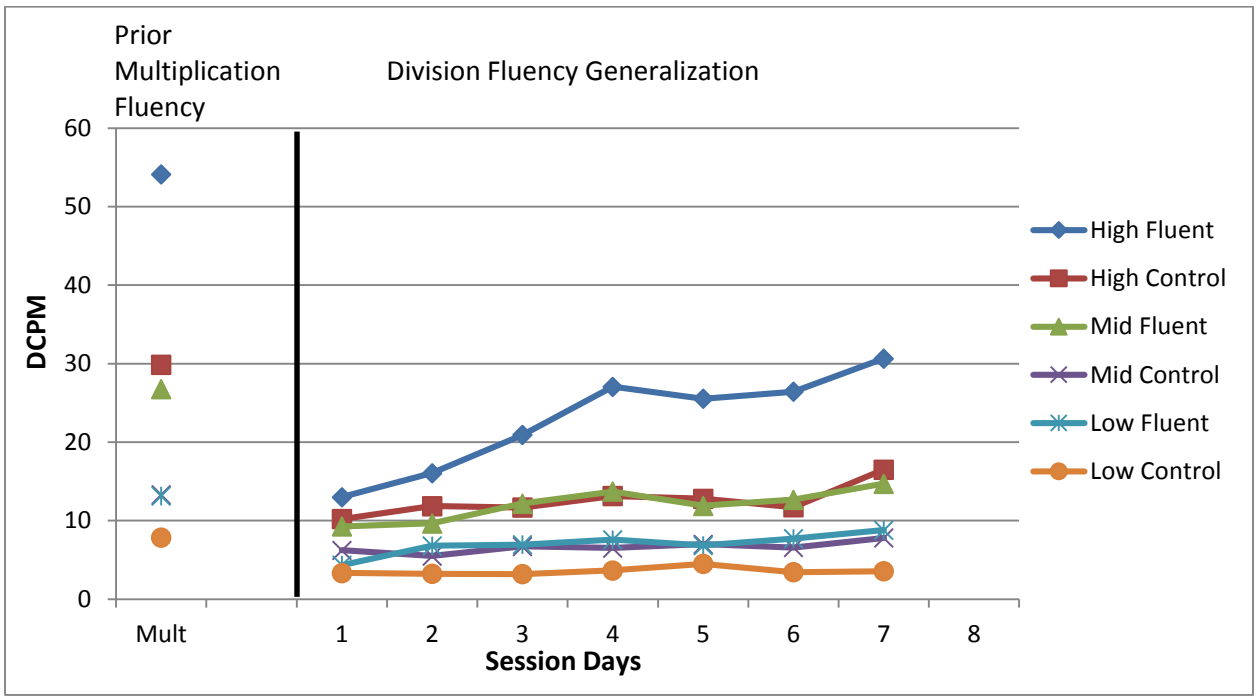


Figure 4.

*Study 2 prior multiplication fluency and division fluency growth in DCPM of student across Group, problem set, and time.*

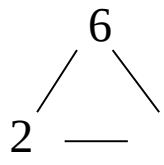
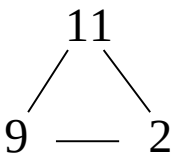
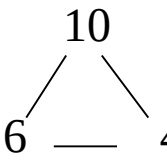
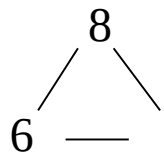
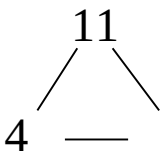
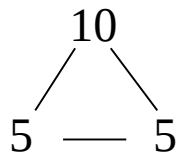
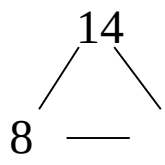
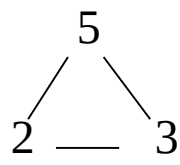
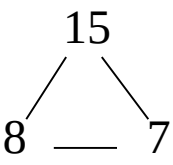
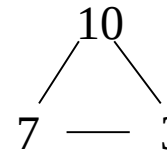
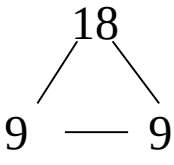
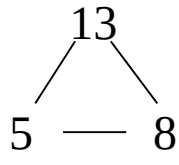
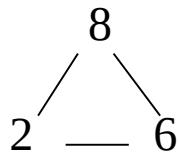
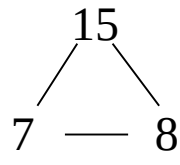
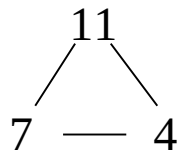
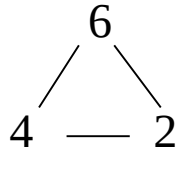
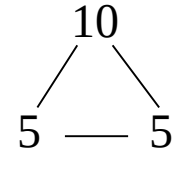
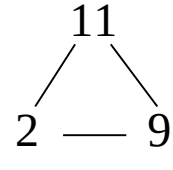


Appendix A
Assessment/ET Probe

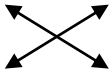
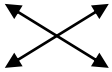
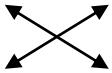
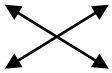
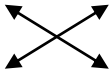
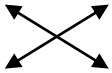
| | | | | | |
|---|---|---|---|---|---|
| 9<br>+ 8 | 3<br>+ 3 | 5<br>+ 2 | 7<br>+ 7 | 9<br>+ 6 | 2<br>+ 8 |
| 5<br>+ 3 | 6<br>+ 7 | 6<br>+ 5 | 3<br>+ 8 | 4<br>+ 4 | 4<br>+ 9 |
| 7<br>+ 7 | 5<br>+ 3 | 6<br>+ 7 | 3<br>+ 3 | 3<br>+ 8 | 9<br>+ 8 |
| 5<br>+ 2 | 4<br>+ 9 | 6<br>+ 5 | 4<br>+ 4 | 9<br>+ 6 | 2<br>+ 8 |
| 9<br>+ 8 | 5<br>+ 3 | 6<br>+ 7 | 3<br>+ 3 | 6<br>+ 5 | 7<br>+ 7 |
| 9<br>+ 6 | 2<br>+ 8 | 5<br>+ 2 | 3<br>+ 8 | 4<br>+ 4 | 4<br>+ 9 |
| 3<br>+ 3 | 9<br>+ 6 | 5<br>+ 3 | 7<br>+ 7 | 3<br>+ 8 | 5<br>+ 2 |
| 6<br>+ 5 | 9<br>+ 8 | 6<br>+ 7 | 4<br>+ 4 | 2<br>+ 8 | 4<br>+ 9 |

Appendix B

CCC-FT

| | | | | | |
|---|---|---|---|---|---|
|  | ○<br>○ |  | ○<br>○ |  | ○<br>○ |
|  | ○<br>○ |  | ○<br>○ |  | ○<br>○ |
|  | ○<br>○ |  | ○<br>○ |  | ○<br>○ |
|  | ○<br>○ |  | ○<br>○ |  | ○<br>○ |
|  | ○<br>○ |  | ○<br>○ |  | ○<br>○ |
|  | ○<br>○ |  | ○<br>○ |  | ○<br>○ |

Appendix C
Think Addition

| | | | |
|---|---|---|---|
| 6<br>- 4 | 2<br>+ 4<br>6  6<br>- 4<br>2 | 6<br>- 4 | Think ?<br>+ 4<br>6 |
| 10<br>- 5 | + 5<br>10  10<br>- 5 | 10<br>- 5 | Think + 5<br>10 |
| 5<br>- 3 | + 3<br>5  5<br>- 3 | 5<br>- 3 | Think + 3<br>8 |
| 8<br>- 2 | + 2<br>8  8<br>- 2 | 8<br>- 2 | Think + 2<br>8 |
| 10<br>- 4 | + 4<br>10  10<br>- 4 | 10<br>- 4 | Think +4<br>10 |
| 15<br>- 8 | + 8<br>15  15<br>- 8 | 15<br>- 8 | Think + 8<br>15 |

Appendix D

Name:_____

| | | | | | |
|---|---|---|---|---|---|
| + 2<br>4 | + 4<br>12 | + 3<br>7 | + 7<br>12 | + 3<br>12 | + 8<br>16 |
| + 5<br>14 | + 7<br>16 | + 5<br>9 | + 7<br>9 | + 6<br>12 | + 6<br>9 |
| + 7<br>16 | + 7<br>9 | + 5<br>9 | + 4<br>12 | + 2<br>4 | + 6<br>9 |
| + 3<br>7 | + 8<br>16 | + 5<br>14 | + 3<br>12 | + 6<br>12 | + 7<br>12 |
| + 4<br>12 | + 5<br>9 | + 6<br>9 | + 3<br>7 | + 2<br>4 | + 7<br>12 |
| + 5<br>14 | + 8<br>16 | + 3<br>12 | + 7<br>16 | + 6<br>12 | + 7<br>9 |
| + 7<br>16 | + 7<br>12 | + 4<br>12 | + 2<br>4 | + 5<br>14 | + 6<br>9 |
| + 3<br>7 | + 3<br>12 | + 5<br>9 | + 8<br>16 | + 6<br>12 | + 7<br>9 |

Appendix E

Assessment Guideline and Checklist

Date_____ Time started_____ Time Completed_____

**Administration Directions**

1. Make sure packets are passed out to **all** students.___

2. Make sure the students have their **first and last name and date** on the back of the packet. Make sure that *no students are working ahead.*_____

3. When students are ready, say **"The packet in front of you has addition problems for you to complete. When I say begin, start working the problems as best as you can. Begin with the first row and work across the page. Do not skip problems! But also do not guess. If you do not know the answer after you try it, you can put an X through it and move on. You will have 1 minute to do as many problems as you can. Is everyone ready?....Begin."**_____

4. Begin timing the students for 1 minute.

5. The students are engaged and on-task._____

6. At the end of 1 minute, say **"stop, put your pencils down."** All students stop working and are not working ahead.____

7. Give the students a short break, ensuring that all students turn to the next two pages of the day's packet to be completed in the same way. _____

Appendix F

Cover, Copy, Compare: Implementation Guidelines & Checklist

Date:_____ Start Time: _____ End Time: _____

1. Pass out the intervention folders to students. Instruct them to take out their CCC packets and write their **first and last name and date** on the back. _____

2. Read the following directions, **"Here are your worksheets. You will have 5 minutes to do as many problems as you can while working on the Cover, Copy, Compare worksheet. If you finish your packet, raise your hand and I will bring you another sheet. Ready. Begin". ____**

3. Stop them after 5 minutes and have them hand in their packets._____

   Remember to constantly be walking around the room to prompt and/or help students who are not working. Ensure students are not looking at their neighbors' work. Look for any mistakes and give students feedback so they can correct any incorrect response(s)._____

Appendix G

Explicit Timing Guideline and Checklist

Date_____ Time started_____ Time Completed_____

1. Pass out the intervention folders to students. Instruct them to take out their ET packets and write their **first and last name and date** on the back. _____
2. When students are ready, say **"The packet in front of you has addition problems for you to complete. When I say begin, start working the problems as best as you can. Begin with the first row and work across the page. Do not skip problems! But also do not guess. If you do not know the answer after you try it, you can put an X through it and move on. You will have 1 minute to do as many problems as you can before we take a break. Is everyone ready?....Begin."_____**
3. Begin timing the students for 1 minute.
4. The students are engaged and on-task._____
5. At the end of 1 minute, say **"stop, put your pencils down."** Make sure all students stop working and are not working ahead. Say **"Put a line after the last problem you completed, and get ready to continue on the next problem.**
6. Look around and make sure all students drew their stopping lines in the appropriate location.____
7. Say **"You will have another minute to do as many problems as you can again before our next break. Remember to start on the very next problem. Is everyone ready?.....Begin."____**
8. Repeat steps 3 through 7 for four more timings.____

Remember to constantly be walking around the room to prompt and/or help students who are not working. Ensure students are not looking at their neighbors' work. Look for any mistakes and give students feedback so they can correct any incorrect response(s)._____

Appendix H

Training Directions CCC Guideline and Checklist

Date_____ Time started_____ Time Completed_____

Step 1: Pass out students folders containing the CCC sheets and instruct them to write their names at the top of the paper.____

Step 2: Read the following directions, "**Today we are going to do something new. We are going to do math problems using something called Cover, Copy, and Compare. Look at your worksheets. On the worksheet you will see a column of fact triangles with one empty space next to it with two small circles in it.___**

Step 3: **Doing Cover, Copy, and Compare with fact triangles is easy. Look at the first triangle. It shows a 5 at the top, a 1 in the bottom left, and a 4 in the bottom right. This means 1 plus 4 equals 5, or 4 plus 1 equals 5. The answer to the two possible problems is always at the top of the triangle, and the two addends are always at the bottom.____**

**When doing Cover, Copy, and Compare with fact triangles, you begin by looking at the triangle and say one of the two related addition problems to yourself. With the first triangle, it shows us that 1plus 4 equals 5, or that 4 plus 1 equals 5. After you say one of the two problems to yourself, cover the triangle with your hand. After you cover it, write the the problem and answers in the space directly next to it. After you have written the problem and answer, uncover the model and check to see if what you wrote is correct. If you have correctly completed the problem, repeat the problem and answer by thinking or whispering to yourself two times as you check off the two small circles. Once you complete this, go to next problem. If you have written the wrong problem or answer, then cross it out and write the correct problem and answer.____**

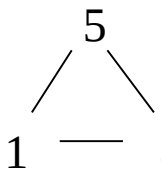
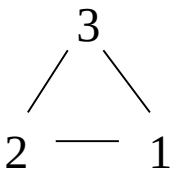
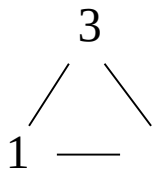
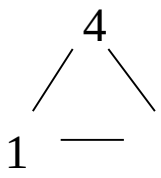
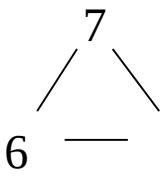
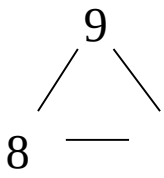
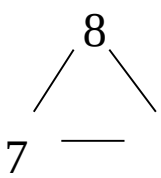
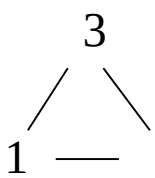
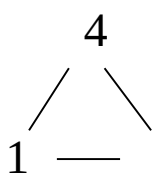
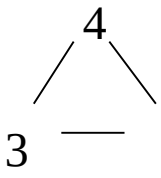
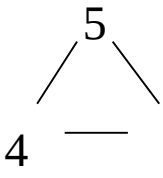
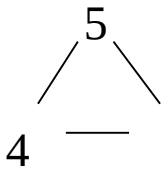
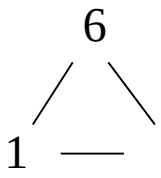
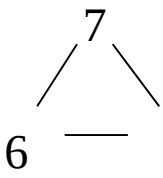
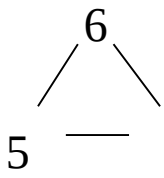
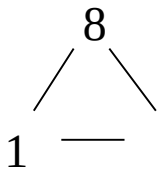
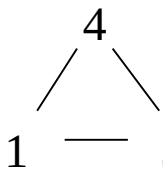
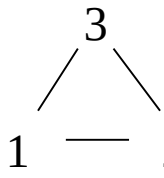
Step 4**: Now let's try the next triangle. It has a 3 at the top, a 2 on the bottom left, and a 1 on the bottom right. So look at the triangle, say one the two addition problems and answers (2 plus 1 equals 3 or 1 plus 2 equals 3) and then cover it. Write your problem and answer. Then uncover the model to see if you did them correctly. Finally, repeat by saying the problem two times while checking off the two small circles. Once completed, go to the next problem._____**

Step 5: **Does everyone understand?....Complete the remaining problems until I say stop.___**

Step 6: Repeat as necessary

Appendix I

CCC-FT Sheet

| | | | | | |
|---|---|---|---|---|---|
|  | |  | |  | |
|  | |  | |  | |
|  | |  | |  | |
|  | |  | |  | |
|  | |  | |  | |
|  | |  | |  | |

Appendix J

Training Explicit Timing Guideline and Checklist

Date_____ Time started_____ Time Completed_____

1. Pass out the intervention folders to students. Instruct them to take out their ET packets and write their **first and last name and date** on the back. _____
2. When students are ready, say **"The packet in front of you has addition problems for you to complete. When I say begin, start working the problems as best as you can. Begin with the first row and work across the page. Do not skip problems! But also do not guess. If you do not know the answer after you try it, you can put an X through it and move on. You will have 1 minute to do as many problems as you can before we take a break. But when I say stop, I would like everyone to put a big line after the last problem they completed. Then instead of turning to the next page, I want you to just start working on the next problem after your line. Is everyone ready?....Begin."_____**
3. Begin timing the students for 1 minute.
4. The students are engaged and on-task._____
5. At the end of 1 minute, say **"stop, put your pencils down."** Make sure all students stop working and are not working ahead. Say **"Put a line after the last problem you completed, and get ready to continue on the next problem, but don't start until I say so. Make your line big enough for me to easily see.**
6. Look around and make sure all students drew their stopping lines in the appropriate location.____
7. Say **"You will have another minute to do as many problems as you can again. Remember to start on the very next problem. Is everyone ready?.....Begin."____**
8. Repeat steps 3 through 7 until all students can accurately complete five timings.____

   Remember to constantly be walking around the room to prompt and/or help students who are not working. Ensure students are not looking at their neighbors' work. Look for any mistakes and give students feedback so they can correct any incorrect response(s)._____

Appendix K

"Think Addition": Implementation Guidelines & Checklist

Date:_____ Start Time: _____ End Time: _____

1. Pass out intervention folders to students. Instruct them to take out their "Think Addition" worksheets. Have them put their names at the top of the paper. _____

2. Read the following directions, **"This worksheet will help us figure out how to do subtraction problems by using what we already know about addition problems. For example, look at the first problem on your page, on the left. It shows the subtraction problem 6-4=___? Think about how we can solve this problem by thinking of addition. Look at the middle box: If you know that 2+4=6, you can reverse the problem to get 6-4=2. So now look at the on the right: If we know this, we can find the answer to 6-4= by thinking what +4=6. Use your addition skills and fill in the subtraction answer on the left.**"____

3. **"Let's look at the next example. It shows the subtraction problem 10-5=____? To figure out what 10-5=____, think about the addition problem of what +5=10? So what +5=10?"____**

    a. If the students do not answer or answer incorrectly, respond by saying **"the answer is 5. You see, 5+5=10. So when we reverse this problem, what is the answer to 10-5=___?"____**
    b. If the students answer incorrectly, repeat step 3.____

    c. If the students do answer correctly, respond by saying **"good job, the answer to 10-5=5"** If students do not respond correctly, refer to step 4.___

**4.** If the students respond correctly, respond by saying **"Good job everyone. Now complete the last 4 problems by yourself. If you have questions, please raise your hand and I will come help you."**

Remember to constantly be walking around the room to prompt and/or help students who are not working or do not understand. Look for any mistakes and give students feedback so they can correct any incorrect response(s). **If students are making mistakes or need help, repeat the lesson from step 3 above. _____**

Appendix L

Cloze Practice Guideline and Checklist

Date_____ Time started_____ Time Completed_____

1. Pass out the intervention folders to students. Instruct them to take out their Cloze packets make sure their **first and last name and date** are on the back. _____
2. When students are ready, say **"The packet in front of you has special problems for you to complete where you need to fill in the missing number rather than finding the addition answer. You need figure out what number is missing that combines with the present number that finishes the fact. Therefore, you will write your answer on top of the problem. The arrows in the first few problems will help you remember where to put your answer. When I say begin, start working the problems as best as you can. Begin with the first row and work across the page. Do not skip problems! But also do not guess. If you do not know the answer after you try it, you can put an X through it and move on. You will have 2 minutes to do as many problems as you can. Is everyone ready?....Begin."_____**

3. Begin timing the students for 1 minute.

4. The students are engaged and on-task._____

5. At the end of 1 minute, say **"stop."** Make sure all students stop working and are not working ahead. Say **"Put a line after the last problem you completed, and get ready to continue on the next problem.**

6. Look around and make sure all students drew their stopping lines in the appropriate location._____

7. Say **"You will have another minute to do as many problems as you can again before our next break. Remember to start on the very next problem. Is everyone ready?.....Begin."____**

8. Repeat steps 3 through 7 for 1 more timings._____

   Remember to constantly be walking around the room to prompt and/or help students who are not working. Ensure students are not looking at their neighbors' work. Look for any mistakes and give students feedback so they can correct any incorrect response(s)._____

VITA

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